

The Number of Atoms above the Photosphere.

IN discussions of intensities of spectral lines the number of atoms producing a line, or the number of atoms above the photosphere, is the most commonly used conception. It may be of interest to determine its meaning and its scope.

We consider first the case of a line produced by interstellar gas, such as a detached Ca^+ line. The 3933 light of the star in passing the consecutive volume elements of space is absorbed by the atoms therein after the formula $dI = -\alpha n \rho I dl = -s \rho I dl$, where α is the atomic absorption coefficient, s the monochromatic absorption coefficient (diffusion coefficient) per unit mass, n and $n\rho$ the number of absorbing atoms per unit mass and per unit volume. The absorbed energy, changed into atomic quantum energy, is re-emitted to all sides; the energy in this way added to the pencil of light considered is imperceptible, because the solid angle filled by the directions in this pencil is extremely small. So no further terms are added to the equation, and the intensity I decreases exponentially: $\log I = -\alpha \int n \rho dl$. It is determined by $\int n \rho dl$, the total number of absorbing atoms contained in a cylinder of cross-section τ extending over the length of the light path. Each successive layer of atoms simply adds its absorbing effect to the preceding layers, and the result is determined by the total number of absorbing atoms. Different points in the line profile, with different α and s , afford the same number. So we may speak here of the number of atoms producing the line. This number is a datum of space between us and the star. The same holds for the case of the B-type companion of ζ Aurigæ, shining through the atmosphere of the K-type component at the end of an eclipse. Here also the pencil of light passing through that atmosphere has a very small angular aperture, so that all the absorbed energy is lost for it.

The matter is different, however, for the ordinary absorption lines produced in stellar atmospheres. Streams of energy are going in all directions through the atmosphere, and the energy absorbed monochromatically from all of them is, by the re-emission in all directions, added to all of them. Nothing is lost, but all is diffused, re-distributed in direction. (If collisions play a perceptible

rôle, then part of this energy is lost for this wave-length, but reappears in other wave-lengths.) The well-known equations for the radiation integrals,

$$dJ/\rho dl = 3(k+s)H, \quad dH/\rho dl = k(J-E),$$

indicate that the net stream of radiation H is obstructed by the continual diffusion (expressed by s), but is increased by the processes of ionization, recombination and collisions (expressed by k), which throw black body radiation into the outgoing stream. The solution of these equations shows that in great depths $J=E$, and $H=k/(k+s) \cdot dE/d\tau$ (the optical depth $\tau \equiv \int k\rho dl$), which is very small for large s . Nearer to the surface J decreases, H increases, both by an exponential term with exponent $\tau\sqrt{(k+s)/k}$, till at the surface they reach their interrelated values* $H=H^\circ\sqrt{k/(k+s)}$ and $J=2H$, where H° is the same quantity for the continuous spectrum near the line ($s=0$), hence $H/H^\circ=r$, the residual intensity. Here we do not have an original continuous spectrum in which the overlying, say Ca^+ , atoms cut out an absorption line. If we could observe the net stream passing through some surface in the depths, we should find a very low value of H , a very dark Ca^+ line. All the layers above it are emitting black body radiation containing this wave-length, and the same layers are continually diffusing and throwing back this wave-length. Looking at the exponential function in the expressions for J and H , we can indicate a certain range of layers (with $\tau\sqrt{(k+s)/k}$ between 0 and say 2 or 3) and say that there the absorption line is formed, or that the Ca^+ atoms in these layers chiefly are forming the line. But it must be borne in mind that in these layers the net stream $H(\lambda)$ is not decreasing, but is increasing from the very small interior value $\sim k/(k+s)$ to the less small surface value $\sim\sqrt{k/(k+s)}$.

The observed residual intensity $r=\sqrt{k/(k+s)}$ (for $s \gg k$ it may be written $\sqrt{k/s}$) does not depend on a total number of atoms. The diffusion coefficient s contains as a factor the number of Ca^+ atoms in unit mass of the atmosphere; in the same way the absorption coefficient k contains the number of atoms taking part in the absorption processes, in the same unit mass. There

* The simplest form, holding for special values of some parameters occurring in the general expressions.

is no objection to multiplying both by the mass contained in a cylinder of unit cross-section extending through all the layers which we found were chiefly producing the line. Then we have in s as well as in k what we may call the total number of atoms producing the line and the total number of atoms producing the continuous absorption. But it does not seem that much is gained by it.

The real quantity determining r , and so the line intensity, is the ratio s/k ; hence the ratio of the number (contained in unit mass) of Ca^+ atoms and that of other atoms producing k . Such ratios are expressed by the concentration of each kind of atoms (fraction of the total number of all the atoms)*. The concentration of atoms producing a certain absorption line consists of the factors: abundance of the element: fraction of this element in the state of ionization considered: fraction of these atoms that is lifted to the energy level by which the line is absorbed: and oscillator strength f for the transition producing the line. The same factors or some of them occur in the expressions for the continuous absorption coefficients.

The use of the number of atoms came up from a former state of theory. In 1905 Schuster considered (among other more complicated cases) the simple model of a purely scattering homogeneous atmosphere above a surface emitting black body radiation. Then the intensity $I(\lambda)$ of the outgoing light at the surface of the atmosphere is $\dagger E(\lambda)/(1+s'h)$, when $E(\lambda)$ is the black body radiation, h is the height of the atmosphere and s' its scattering (or diffusion) coefficient for unit volume. In 1927 Unsöld, in applying the physical formula for the diffusion coefficient in the resonance wings to the profile of Fraunhofer lines, especially H and K, made use of this Schuster formula. The diffusion coefficient in these wings is given, per atom, by $\sigma f/\Delta\lambda^2$, where σ is calculable from physical constants. We find s' by multiplying this expression by the number of atoms per unit volume; hence the product $s'h = \sigma f N/\Delta\lambda^2$, where N denotes the number of Ca^+ atoms in the atmosphere above unit surface of the deep emitting surface. So the residual intensity $r = I/(1+s'h)$ is immediately connected with the

* The fundamental importance of this "concentration" was pointed out by Milne (*M. N.* 89, 18, 1928).

† More accurately, the denominator is $1 + \frac{3}{2}s'h$, but Schuster's less accurate form is the one usually employed.

number of atoms producing it by their monochromatic absorption. By connecting this model with the old Kirchhoff theory of a photosphere, emitting a continuous spectrum, and an overlying atmosphere producing the Fraunhofer lines, this number was also spoken of as the number of atoms above the photosphere (*i. e.*, above unit surface of it). Unsöld took the half width $\Delta\lambda$ of the lines H and K at the points of their profile where $r=1/2$; then $s'h=1$, and from the known values of f , σ , and $\Delta\lambda$ the number N was computed.

Although in later years better expressions for the residual intensity in a Fraunhofer line were given by Eddington and Milne, the use of the Schuster formula was continued by other astrophysicists, because it afforded in an easy way a datum for the star from the profile of a line. It was indeed easy, but the trouble was that the formula that was correct for the model assumed by Schuster was not applicable to the stellar atmosphere according to modern conceptions. This is directly clear if we compare the results of both formulæ for other points of a line profile and express them relatively to those for $r=1/2$.

For	$\Delta\lambda=4/\sqrt{3}$	1	1/3	1/10
s' or s' (relative)	$=3/16$	1	9	100
$r=\sqrt{\frac{k}{k+s}}$ (for $k=1/3$)	$=0.8$	1/2	1/5.3	1/17.4
$\frac{1}{1+s'h}$ (for $h=1$)	$=0.84$	1/2	1/10	1/101
N comp. from r	$=4/3$	1	4.3/9	16.4/100.

For $s \gg k$ the Schuster formula gives r decreasing as the inverse of s , the correct formula as the inverse square root of s *. If Unsöld's computation had been made for other points of the profile other values of N would have been found, the smaller as r is taken smaller. The exponential term in H, the net stream, shows that for the darker parts of a line (small r and large s') the layers in which the line is formed are only a narrow upper part of the atmosphere. If we try to represent this process of formation by a Schuster model then the "photosphere" emitting the black radiation should be

* If k is assumed to vary proportionally to the pressure, then r is found to be proportional to the inverse cube root of s (*M. N.* 91, p. 146).

placed at a small depth, with a low atmosphere above it, and a smaller "number of atoms" is found to produce these darker parts of the line.

So it appears that we cannot speak of a number of atoms producing a certain absorption line. Each part of the line, in this form of expression, is produced by a different number of atoms, the darker central parts by a smaller number than the less darkened outer parts. This does not mean, of course, that the computation of this N has no importance. When for different lines the half width for the same $r=1/2$ is measured, the resulting values of N are proportional to the concentrations of the active atoms (taking account, of course, of variations of k and other quantities with λ), and so are valuable for deriving the composition of the stellar or solar atmosphere. Their meaning, however, is complicated and depends upon the peculiarity of the observational data themselves.

Milne has made use of this number of atoms producing a line for the inverse problem to derive line intensities from theoretical assumptions*. He introduces the optical depth $\tau = \int k \rho dl$, and he computes at what optical depth a radiating surface in Schuster's model must be placed in order to give the same residual intensity r as follows from the correct formula. He finds $\tau = r/(1+r)$, corresponding to our table above, hence $\tau = 1/3$ for $r = 1/2$. This was computed in the supposition of constant k and s throughout the atmosphere. Assuming now that it holds also for the case of k and s variable with pressure and depth in the atmosphere, he computes the number N of active atoms above the surface defined by $\int k \rho dl = 1/3$. Then, because $s'h$ in Schuster's formula $= 1$, the value $\Delta\lambda = \sqrt{\sigma f N}$ determines the half width of the line for $r = 1/2$, which is a direct measure of the line intensity. The variations of these N with temperature and gravity measure the variations of line intensity over the spectral classes and groups. Here N is simply a substitute for line intensity. It is not intended to give information about the star; but it tells something

* "Theoretical contours of absorption lines" (*M. N.* 89, 3, 1928).
The more accurate formula $\tau = \frac{4}{3} \frac{r}{r+1}$ was derived in *Phil. Trans. Roy. Soc.* 228 A, 434 (1929).

about the spectral line. By a simple numerical operation it is connected with the half width of the line.

The same holds for Russell's computations * of N by means of improved values of the absorption coefficient; it is an expression of line intensity, and its variations show the variations of line intensities. When, however, conversely, the total number of atoms N is derived from measured line intensities, this result only gives the observational datum in another form. When first Stewart and then Unsöld made the reduction to this form, it was an important step. But we cannot stop here. Our aim is to get data about the star, the stellar atmosphere. The number N is not a datum about the star. The concentration of active atoms, of atoms producing a certain line, is a datum about the star. If we wish to find a real physical datum about the star from a measured line profile we derive first s/k (or the parameter determining variable values of s/k) from r , then take k for the conditions of this atmosphere, and then apply the relation $s = \text{concentration of active atoms} \times \text{functions of physical quantities including } \sigma, \times 1/\Delta\lambda^2$ (if we include the oscillator strength f in this concentration). In the most simple suppositions (s and k constant) this computation is quite as simple as a computation of N , and it has a direct meaning for the stellar atmosphere.

Russell introduced the use of the oscillator strengths f within a multiplet to determine the relative abundance of different elements in the solar atmosphere. If line intensities (either expressed by estimated Rowland numbers, or by measured equivalent widths) are set against the values fN in a diagram (N constant within each multiplet), then from this diagram we can read off the numbers N for different levels of different elements. It is clear that in this way only relative values can be obtained; some values of N must be taken from other sources. This value in fact is arbitrary, and the resulting other values have no further meaning for the star than the N considered above. Exactly the same procedure could have been followed when speaking of real data of the stars—the concentrations. They contain the f as factors; so in a diagram these concentrations are set against line intensities (this diagram is identical with that of N , but for the name of the scale unit), and

* "Opacity Formulæ and Line Intensities" (*Ap. J.* 78, 239, 1933).

then for different elements the concentrations may be read off from this diagram. Of course, here also the resulting values are relative values, and they can be made absolute only by the knowledge of the concentration of at least one element in one level.

ANT. PANNEKOEK.

C O R R E S P O N D E N C E.

To the Editors of 'The Observatory'.

Theoretical Colour Temperatures.

GENTLEMEN,—

Dr. Unsöld, in his letter in the August number of *The Observatory*, is right in remarking that in my paper on "Theoretical Colour Temperatures" his paper in the *Zeitschrift für Astrophysik* should have been quoted as well as Dr. Biermann's papers. I must apologize for not reading his treatment of this problem. The computations on line intensities, which had to be ready by a definite date, so entirely occupied my time last year that I had to neglect the regular study of current publications. When I saw the results of Messrs. Greaves, Davidson and Martin, and realized that my computed absorption coefficients could clear up the matter, I sent a note about it, without looking for similar investigations.

It is certainly desirable, in general, to give close attention to all current literature; but with the enormously increasing productivity of science, this can be done only at the cost of time which should be spent on one's own research work. It is often easier and more efficient to attack a problem in one's own way than to follow another's development of the same problem. Independent treatments of the same problem are more often no waste of scientific energy.

I am, Gentlemen,
Yours faithfully,

University of Amsterdam,
1935 October 20.

ANT. PANNEKOEK.