LIMB DARKENING.

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In my paper on "Theoretical Colour Temperatures" * I pointed out how the variation of the absorption coefficients of a stellar atmosphere with wave-length and temperature determines a deviation of the so-called colour temperatures from the real (effective) temperatures. It is easily seen that it has also an important bearing upon the coefficient of limb darkening.

The intensity emitted at the surface of the atmosphere in any direction, making an angle θ with the external radius, is determined by

$$I_0 = \int_0^\infty E(\tau) e^{-\tau \sec \theta} \sec \theta d\tau,$$

where τ is the optical depth, and the black body radiation at this depth τ is given by

$$E(\tau) = E_0(1 + \tfrac{3}{2} c\tau \bar{k}/k) = E_0(1 + C\tau).$$

The result of the integration is

$$I_0 = E_0(1 + \tfrac{3}{2} c\bar{k}/k \cos \theta) = E_0(1 + C \cos \theta).$$

Then the coefficient of limb darkening

$$u \equiv \frac{I(0) - I(90)}{I(0)} = \frac{C}{1 + C}$$

is seen to depend in a very simple way upon this $C \equiv \tfrac{3}{2} c\bar{k}/k$.

For the total radiation, if C is taken simply 3/2 we have $u = 0{\cdot}60$, as was first deduced by Milne.† If we take account of c only, which expresses the bluer colour of the radiation of the deeper layers, we find u increasing from 0·60 for λ 6000 A. to 0·69 for λ 4000 A., hence there is a reddening of the limb. If now we take account of the factors $\bar{k}/k$ the limb darkening is found to vary with temperature too.

We take the values of C from Table 26 in Publ. No. 4 of the Amsterdam Astronomical Institute (where C depends on gravitation, *i.e.* for temperatures 9500 – 5000°, the value for the solar gravity $\log g = 4{\cdot}4$ is taken). Then we find the results given in the following table.

* *M.N.*, **95**, 529, 1935.
† *M.N.*, **81**, 371, 1921.

[TABLE

T	C 6000 A.	5000 A.	4000 A.	u 6000 A.	5000 A.	4000 A.
25200°	0·03	0·06	0·13	0·03	0·06	0·12
16800°	0·09	0·17	0·40	·08	·15	·29
12600°	0·30	0·60	1·38	·23	·38	·58
10080°	0·69	1·38	3·3	·42	·58	·77
8400°	1·00	2·00	4·8	·50	·67	·83
7200°	1·44	2·35	4·0	·59	·70	·80
6300°	2·05	2·45	2·25	·67	·71	·69
5600°	2·35	2·25	1·55	·70	·69	·61
5040°	2·35	2·00	1·20	·70	·67	·55
4200°	2·05	1·41	0·62	·67	·60	·38

The variations of limb darkening with temperature and wave-length, appearing in these values, can be expressed in the following qualitative form. For B-type stars the limb darkening is small. For A-type stars it is very large in the blue, strongly decreasing to the larger wave-lengths. For stars of advanced type, G–K, the limb darkening is smaller for blue than for yellow rays, so that the darkened limb of these stars must appear more bluish than the centre. For the Sun we may deduce the limb darkening from the values of the limb-intensity (for $\sin\theta = 0{\cdot}95$, $\cos\theta = 0{\cdot}312$) given by Milne * after a table computed by Lindblad.† We have for

λ	3737	4265	5062	5955	6702
limb-intensity	0·432	0·445	0·529	0·595	0·640 (centre = 1)
u	0·83	0·81	0·68	0·59	0·52

Hence for 6000 A., 5000 A., 4000 A. the observed u is nearly 0·59, 0·69, 0·82. So here we have the same discrepancy as was noted for the colour temperature; solar phenomena correspond to those computed theoretically for a temperature of over 7000°. This indicates that the computed absorption coefficients need still some correction.

These results may have some importance for the eclipsing variables, the only stars where we can find observational evidence about limb darkening. It will be interesting to test the theoretical values, especially the strong dependence on wave-length, by accurate photoelectric measures of the appropriate eclipsing variables in their minima.

* *Thermodynamics of the Stars*, p. 149; *Handbuch der Astrophysik*, III, 1.
† *Nova Acta Upsala*, **6**, i, p. 17, 1923.