of the photosphere, which will lift the ionized *Ba* atoms, by absorption of λ 4934 or 4554 into one of the $2P$ states; then it appears probable that the $Ba+$ atom in this state will be less likely to recombine than one in the S state. For sodium this influence must be very much smaller, in the first place because the enhanced lines of sodium lie in the extreme ultraviolet, beyond λ 377, where the radiation of the photosphere is probably insensible, so that the sodium atom, once ionized, is not further disturbed by sunlight (*Astroph. Journ.* 55, 356—57).

Now our analysis shows that this explanation of the discrepancy cannot be right. The existence of higher quantum states of the ionized atom in the case of thermal equilibrium certainly increases the rate of ionization by the well known factor $b'(T) = q_1 + q_2\, e^{-(\chi_1 - \chi_2)/kT} + \ldots\ldots$; but the second term is far too small to have any sensible influence. In the case of photospheric radiation our formula shows that the case is hardly different, the exponential expression being $1/370$ instead of $1/498$. Moreover the formula shows that the deviation from the isothermal case becomes greater for smaller wavelengths, because the anisotropic radiation from below has the short wavelength side of the spectrum stronger, the long wavelength side fainter, than an isotropic radiation of the same total intensity; thus the $Na+$ atoms with lines in the far ultraviolet must show the influence of the photospheric radiation even more strongly than the $Ba+$ atoms.

On the images by an irregular coarse grating, by *A. Pannekoek.*

Extrafocal images formed by a coarse grating before the objective have been proposed by HERTZSPRUNG as an expedient for fundamental photographic photometry. The rather large irregularities found by measurement in the grating used for photometric work at the Bosscha Observatory, Lembang, showed the desirability of investigating the influence of such irregularities on the photometric results.

We suppose only variations in one dimension x; physically this corresponds to strict parallelism of all the limits between the dark and bright spaces, and to a rectangular aperture. The average breadth of the dark spaces is d, of the bright spaces l; the average value of a period is $L = l + d$; their number is n. The deviations of the real limits from an ideal grating, where the breadth is everywhere exactly d and l, are $e_1\, e_2 \ldots e_{2n}$; then by our definitions $e_1 + e_2 + \ldots e_{2n} = 0$ $e_1 - e_2 + e_3 - e_4 + \ldots - e_{2n} = 0$.

If we put for the amplitude integrals $\int \cos 2\pi \frac{x\alpha}{\lambda}\, dx = C$ and $\int \sin 2\pi \frac{x\alpha}{\lambda}\, dx = S$, the integrals being taken over the whole of the bright spaces, then the intensity at a distance αf from the central image will be given by $I = C^2 + S^2$. For the first diffraction image $\alpha_1 = \lambda/L$. Putting $2\pi \frac{\alpha_1 x}{\lambda} = 2\pi \frac{x}{L} = \varphi$, we have $C = \frac{L}{2\pi}\int \cos\varphi\, d\varphi$, $S = \frac{L}{2\pi}\int \sin\varphi\, d\varphi$. Putting $\frac{1}{2}(l-d)/L = \psi/\pi$ and $2\pi\, e/L = \varepsilon$, the limits of the integrals are

$-\psi + \varepsilon_1$ to $\pi + \psi + \varepsilon_2$; $2\pi - \psi + \varepsilon_3$ to $3\pi + \psi + \varepsilon_4$; ... etc., and the integrals become

$$C = \frac{L}{2\pi}\left\{ + \sin(\psi - \varepsilon_1) - \sin(\psi + \varepsilon_2) + \sin(\psi - \varepsilon_3) - \sin(\psi + \varepsilon_4) + \ldots \right\}$$

$$= \frac{L}{2\pi} \sin\psi \left\{ \cos\varepsilon_1 - \cos\varepsilon_2 + \cos\varepsilon_3 - \cos\varepsilon_4 + \ldots \right\} - \frac{L}{2\pi}\cos\psi \left\{ \sin\varepsilon_1 + \sin\varepsilon_2 + \ldots \right\}$$

$$S = \frac{L}{2\pi}\left\{ - \cos(\psi - \varepsilon_1) - \cos(\psi + \varepsilon_2) - \cos(\psi - \varepsilon_3) - \cos(\psi + \varepsilon_4) - \ldots \right\}$$

$$= -\frac{L}{2\pi}\cos\psi \left\{ \cos\varepsilon_1 + \cos\varepsilon_2 + \cos\varepsilon_3 + \ldots \right\} - \frac{L}{2\pi}\sin\psi \left\{ \sin\varepsilon_1 - \sin\varepsilon_2 + \ldots \right\}$$

If the deviations ε are not large, such that ε^3 may be neglected, while the second term of each formula disappears because $\Sigma\,\varepsilon = 0$ and $\Sigma\,(\varepsilon_1 - \varepsilon_2) = 0$, then

$$C = \frac{L}{2\pi}\,\frac{\sin\psi}{2}\left\{ -\varepsilon_1^2 + \varepsilon_2^2 - \varepsilon_3^2 + \ldots \right\}$$

$$S = -\frac{L}{2\pi}\cos\psi \left\{ 2n - \tfrac{1}{2}(\varepsilon_1^1 + \varepsilon_2^2 + \ldots) \right\}$$

In the intensity $I = C^2 + S^2$ the first term becomes of the 4th order and unless $\cos\psi$ is very small, i. e. the wires are very thin compared with the bright spaces, may be omitted. Putting $\Sigma\,\varepsilon^2 = 2\,n\,\mu^2$ (thus μ being the mean value of the deviations) we get

$$S = -\frac{n\,L}{\pi}\cos\psi\,(1 - \tfrac{1}{2}\mu^2) \text{ and}$$

$$I = \frac{n^2 L^2}{\pi^2}\cos^2\psi\,(1 - \mu^2).$$

In this deduction it is not supposed that the deviations e and ε behave as accidental errors; they may show any systematic course up and down.

The intensity at the central image is found in the same way (cos φ being 1) $I_c = n^2 l^2$, and the intensity without grating $I_0 = n^2 L^2$. Thus

$$I/I_0 = 1/\pi^2 . \cos^2 \psi (1-\mu^2) \text{ and } I_c/I_0 = l^2/L^2 = (1/_2 + \psi/\pi)^2.$$

For the second and the higher diffraction images ($\alpha = 2\,\lambda/L$, $3\,\lambda/L$, etc.) the limits of the integrals become twice, 3 times, etc. the former values; we find

$$I_2/I_0 = 1/\pi^2 . \sin^2 2\psi (1-4\mu^2);\; I_3/I_0 = 1/\pi^2 . \cos^2 3\psi (1-9\mu^2).$$

For equal breadth of the bright and the dark spaces I_2 vanishes. The brightness of all these diffraction images is diminished in consequence of the irregularities in the breadth of the dark and the bright spaces.

Since, however, the light that is lacking in these images, must be dispersed somewhere else in the focal plane beside them, it may be that part of it is gathered up in the extrafocal images. The brightness in the centre of an extrafocal image is determined by the sum total of the light falling in the focal image within a circle of the size of the extrafocal image around this centre. Thus the distribution of the light in the focal plane (coordinate α) must be determined.

We consider a point at the outer (or the inner) side of the first diffraction image at distance $\frac{r}{n}\alpha$; then the phase angle $2\pi\frac{\alpha x}{\lambda} = \left(1+\frac{r}{n}\right)\varphi$, and the limits of the integrals over the bright spaces are

$$-\psi + \varepsilon_1,\; \pi\left(1+\frac{r}{n}\right) + \psi + \varepsilon_2,\; 2\pi\left(1+\frac{r}{\pi}\right) - \psi + \varepsilon_3, \text{ etc.}$$

We will take $\psi = 0$ in order to avoid complicated formulas. The integrals now become

$$C_r = -\frac{L}{2\pi}\left\{\sin\varepsilon_1 + \sin\left(\frac{r}{n}\pi + \varepsilon_2\right) + \sin\left(\frac{2r}{n}\pi + \varepsilon_3\right) + \ldots\right\}$$

$$S_r = -\frac{L}{2\pi}\left\{\cos\varepsilon_1 + \cos\left(\frac{r}{n}\pi + \varepsilon_2\right) + \cos\left(\frac{2r}{n}\pi + \varepsilon_3\right) + \ldots\right\}$$

Since there are $2n$ terms, the periodic argument goes r times through the circumference 2π.

We suppose the deviations ε small; thus the formulas may be written, neglecting the terms with ε^2:

$$C_r = -\frac{L}{2\pi}\left\{\varepsilon_1 + \varepsilon_2\cos\frac{r\pi}{n} + \varepsilon_3\cos\frac{2r\pi}{n} + \ldots\right\}$$

$$S_r = +\frac{L}{2\pi}\left\{\varepsilon_2\sin\frac{r\pi}{n} + \varepsilon_3\sin\frac{2r\pi}{n} + \ldots\right\}$$

Now the $2n$ values $\varepsilon_1 \ldots \varepsilon_{2n}$ can always be represented by a Fourier series

$$\varepsilon_{s+1} = a_0 + a_1\cos\frac{s\pi}{n} + a_2\cos\frac{2s\pi}{n} + \ldots a_{n-1}\cos\frac{(n-1)s\pi}{n}$$
$$+ b_1\sin\frac{s\pi}{n} + b_2\sin\frac{2s\pi}{n} + \ldots b_{n-1}\sin\frac{(n-1)s\pi}{n}$$

Introducing these values in the formulas for C_r and S_r the coefficient of each a or b becomes a series of trigonometric products whose sum total is zero, except for a_r and b_r where it is a sum of $2n$ squares of sinus or cosines, evenly distributed over the circumference.

The value of this sum is n; thus we have $C_r = -\frac{L}{2\pi} n\, a_r$ and $S_r = +\frac{L}{2\pi} n\, b_r$ and the intensity at this point $I_r = \frac{L^2}{4\pi^2} n^2 (a_r^2 + b_r^2)$.

Thus proceeding from the place of the first diffraction image to that of the second one with n equal steps, we find intensities just corresponding to the squares of the amplitudes of the consecutive Fourier terms up to the nth, the mechanism of diffraction performing here the harmonic analysis of the ε values. The same values we find at the other side, from the first diffraction image towards the central one. The diffraction figure of the central image and each of the others, caused by the whole aperture, corresponds in size to one of these steps. Thus it is easily seen that between the points taken above the intensities have intermediate values.

If we are able to collect the sum total of these intensities into one image, it will have the brightness

$$\Sigma I_r = 2 \times \frac{L^2}{4\pi^2} n^2 (\Sigma a_r^2 + \Sigma b_r^2).$$

Now we have $$\Sigma \varepsilon^2 = n(\Sigma a^2 + \Sigma b^2);$$

thus $$\Sigma I_r = \frac{L^2}{4\pi^2} 2n \Sigma \varepsilon^2 = \frac{n^2}{\pi^2} L^2 \mu^2.$$

This is exactly the amount by which I was diminished in consequence of the irregularities; thus in such an image extending from the centre to the place of the second diffraction image the whole theoretical intensity would be collected, just as if there were no irregularities.

But we are not able to collect all this light into a single image. In applying this method the extrafocal central and first diffraction images are just separated; thus the irregularly dispersed light is only gathered up at most as far as $1/_2\, n$; the higher coefficients $a_{1/_2 n}$ to a_n and $b_{1/_2 n}$ to b_n are even contributing to the extrafocal central image.

We can make an estimate of the values of these two groups of terms by computing the means and the differences of every two consecutive values of ε:

$$\varepsilon_s = \Sigma\,(a_r \cos r\,s\,\varphi + b_r \sin r\,s\,\varphi);$$
$$\varepsilon_{s+1} = \Sigma\,(a_r \cos r\,(s+1)\,\varphi + b_r \sin r\,(s+1)\,\varphi)$$
$$\tfrac{1}{2}\,(\varepsilon_{s+1} + \varepsilon_s) = \Sigma\, a_r \cos\,(s + \tfrac{1}{2})\,r\,\varphi \cos \tfrac{1}{2}\,r\,\varphi + \Sigma\, b_r \sin\,(s+\tfrac{1}{2})\,r\,\varphi \cos \tfrac{1}{2}\,r\,\varphi$$
$$\tfrac{1}{2}\,(\varepsilon_{s+1} - \varepsilon_s) = \Sigma\, a_r \sin\,(s + \tfrac{1}{2})\,r\,\varphi \sin \tfrac{1}{2}\,r\,\varphi + \Sigma\, b_r \cos\,(s+\tfrac{1}{2})\,r\,\varphi \sin \tfrac{1}{2}\,r\,\varphi.$$
$$\Sigma \left\{\frac{\varepsilon_{s+1} + \varepsilon_s}{2}\right\}^2 = n\,\Sigma\,(a_r^2 + b_r^2)\cos^2 \tfrac{1}{2}\,r\,\varphi$$
$$\Sigma \left\{\frac{\varepsilon_{s+1} + \varepsilon_s}{2}\right\}^2 = n\,\Sigma\,(a_r^2 + b_r^2)\sin^2 \tfrac{1}{2}\,r\,\varphi.$$

Here the square amplitudes $(a^2 + b^2)$ are multiplied with factors, which for $r = 1$ to n decrease from 1 to 0 for the means, increase from 0 to 1 for the half differences. Separating them into the groups $r = 1$ to $\tfrac{1}{2}\,n$, and $\tfrac{1}{2}\,n$ to n, the coefficients in these groups are for the means 1 to $\tfrac{1}{2}$, (average 0.82) and $\tfrac{1}{2}$ to 0 (av. 0.18), for the half differences 0 to $\tfrac{1}{2}$ (average 0.18) and $\tfrac{1}{2}$ to 1 (average 0.82). Thus putting

$$\Sigma \left(\frac{\varepsilon_{s+1} + \varepsilon_s}{2}\right)^2 = 2\,n\,(0.82\,\mu_1^2 + 0.18\,\mu_2^2)$$
$$\Sigma \left(\frac{\varepsilon_{s+1} - \varepsilon_s}{2}\right)^2 = 2\,n\,(0.18\,\mu_1^2 + 0.82\,\mu_2^2),$$

$2\,\mu_1^2$ may be taken for $\sum_{0}^{\frac{1}{2}n} (a^2 + b^2)$, and $2\,\mu_2^2$ for $\sum_{\frac{1}{2}n}^{n} (a^2 + b^2)$.

Then the light collected in the first diffraction image will be

$$I = \frac{n^2 L^2}{\pi^2}\left\{\cos^2 \psi\,(1 - \mu^2) + \mu_1^2\right\}$$

and the central image $I_c = n^2\,l^2 + \dfrac{n^2\,L^2}{\pi^2}\,\mu_2^2$; thus

$$I/I_0 = \frac{1}{\pi^2}\left\{\cos^2 \psi\,(1 - \mu^2) + \mu_1^2\right\} \quad I_c/I_0 = l^2/L^2 + \mu_2^2/\pi^2.$$

Since as a rule the large values of ε will appear in the longer waves, we may expect that μ_1^2 will not differ very much from μ^2, and μ_2^2 will be small. But only exact measures of the grating can decide whether the deviation from the simple theory is relevant.