groupings are not prominent in the α Cygni spectrum. In discussing this question, Mr. Stratton makes the suggestion that as the first spectrum was of A type, it may be possible that we have here, in the ultra-violet, traces of an intermediate stage, approximating to G type, due to the superposition of arc absorption lines on the enhanced lines. No explanation is offered at present of this abnormality.

The Distance of the Milky Way. By Dr. A. Pannekoek.

(*Communicated by Dr. W. de Sitter.*)

1. By several modern researches formulæ have been deduced for the numbers of stars per square degree which depend on two variables, the limiting magnitude and the galactic latitude, thus: $N(m, l)$. These formulæ define our star system as a figure of revolution, in which the star density depends on distance and galactic latitude. In the direction of the galactic poles the density is least, and diminishes rapidly outwards; in the galactic plane the star density also decreases regularly with increasing distance, but at a slower rate. They represent our sun as lying in the midst of a flat star cluster whose densest parts measure some 1000 parsecs.

This character of our star system is not in accordance, however, with the appearance of the Galaxy. We see the Milky Way as a belt of luminous clouds, patches, and drifts, divided by less luminous regions or dark gaps and rifts. If we go in the direction of such a star-cloud, the star density, after we have left our central cluster, must increase at first on the nearer side of the centre of the cloud, and decrease on the farther side. The aspect of the Milky Way shows that by treating the galactic zone as a whole we intermingle parts of the universe of a great diversity of structure, and thus obscure the special character of this structure, viz. the aggregation of stars in clouds, separated by regions agreeing perhaps with the galactic poles. In studying the distribution of stars in our universe we must treat the different parts of the Galaxy, especially the great star-clouds and streams, separately, determine the function $N(m)$ for each, and derive from it the variation of star density with distance.

2. In a paper printed in the *Proceedings of the Amsterdam Academy*, 1910 June, a first attempt was made to find the function $N(m)$ for some parts of the Milky Way.* The parts chosen were the southern part of the great bright elliptic patch between β and γ Cygni, the poorer regions east of it, and a part of the Eastern stream between 10° and 20° declination, in Aquila and

* "Researches into the Structure of the Galaxy" (*Proc. Amst. Acad. S* i., 1910, p. 239).

Sagitta, including the bright patch of χ Aquilæ. The sources used for the numbers of stars of various magnitudes were:—

(1) The *Bonner Durchmusterung* for the magnitudes 6·5, 8, and 9 (limiting values of m for the three regions investigated 6·55, 6·56, and 6·56; 8·07, 8·08, and 8·08; 9·19, 9·22, and 9·24).

(2) The gauges of Sir William Herschel, which are especially numerous in these regions ($m = 13·90$).

(3) The star counts (as yet unpublished) made by Th. Epstein at Frankfort in the years 1877–1888 with a 6-inch telescope ($m = 12·51$).

(4) The *Carte du Ciel*, which furnishes two sorts of data, a brighter limit being provided by the Catalogue plates, a fainter one by the Chart plates. For the Cygnus regions we could only make use of the Potsdam Catalogue plates ($m = 11·73$); for the Aquila stream we had catalogue and chart plates of the Bordeaux zones +14°, +16°, +17° ($m = 11·73$ and 13·20).

The limiting magnitudes given in parentheses were found from the star numbers themselves. For a number of regions distributed over the whole sky, or at least over different galactic latitudes, the values of N were deduced, and with the aid of Kapteyn's table in *Groningen Public.*, 18, which gives log N as a function of the limiting magnitude and the galactic latitude, a number of values for m were derived. For particulars of the values adopted, the former paper may be referred to. In *Groningen Public.*, 28, Dr. Van Rhijn has since given corrections to reduce these magnitudes (based on the scale of Parkhurst) to the Harvard Scale. Using these corrections, the results of Table I. are found.

TABLE I.

	Cygnus Bright Patch.			Cygnus Faint Region.			Aquila Stream.		
	m.	log N.	$\frac{d \log N}{dm}$.	m.	log N.	$\frac{d \log N}{dm}$.	m.	log N.	$\frac{d \log N}{dm}$.
B.D.	6·60	9·46		6·61	9·40		6·61	9·45	
,,	8·15	0·352	0·58	8·16	0·346	0·61	8·16	0·265	0·53
,,	9·29	1·031	0·60	9·32	1·020	0·58	9·34	0·898	0·54
C. d. C. Cat.	11·97	2·260	0·46	11·97	2·176	0·44	11·97	2·249	0·51
Epstein	12·89	2·629	0·40	12·89	2·513	0·37	12·89	2·557	0·33
C. d. C. Chart	...	...	0·62	...	...	0·38	13·76	3·205	0·74
Herschel	14·67	3·737		14·67	3·182		14·67	3·948	0·82

The values of the gradient $\frac{d \log N}{dm}$ show clearly the influence of a remote star-cloud; after the normal decrease, corresponding to the regular decrease of star density in the central cluster, the gradient increases for stars beyond the 13th magnitude. In the

Aquila stream this increase is more strongly marked than in the Cygnus patch. In the Cygnus region east of the bright patch, filled with faint light, the number of stars up to the 12th magnitude is nearly the same as in the bright patch itself; but the increase beyond the 13th magnitude is absent. The appearance of the Milky Way in this region also gives the impression of a bright star-cloud rising rather suddenly from the faint background on its eastern side.

3. To find theoretically the number of stars of magnitude m produced by such a star-cloud we assume the luminosity function as derived by Kapteyn,

$$\log \phi(i) = \text{const} - \frac{1}{\alpha^2}(M - M_0)^2.$$

For the distance r we introduce a new variable: the modulus of distance ρ, defined by

$$\rho = 5 \log r \quad (\rho = 0 \text{ for } \pi = 0''\cdot 1).$$

If we suppose that in a star-cloud at distance r_0 the density decreases on both sides according to the law

$$\log \Delta = \text{const} - \frac{1}{\beta^2}(\rho - \rho_0)^2,$$

then the number of stars of magnitude m is given by

$$A_1(m) = \int_{-\infty}^{+\infty} \Delta(\rho)\, 10^{0\cdot 6\rho - 1/\alpha^2 (m - M_0 - \rho)^2} d\rho,$$

or

$$\log A_1(m) = \text{const} - \frac{1}{\alpha^2 + \beta^2}\{m - (\rho_0 + M_0 + 0\cdot 3\beta^2)\}^2.$$

Now, if an agglomeration of stars appears to us as a cloud of 15° or 20° diameter, its extension along the line of sight being not greater, the constant β^2 must be < 1, whereas α^2 is nearly 30. Thus in this formula we may neglect wholly the extension of the star-cloud in depth, as its influence on the distribution of A_1 is insignificant compared with the dispersion caused by the luminosity curve. We will assume, therefore, an agglomeration of stars, lying all at the distance r. Then we have

$$\log A_1(m) = \text{const} - \frac{1}{\alpha^2}(m - m_0)^2,$$

where $m_0 = M_0 + \rho$.

To this A_1 must be added the number of stars, $A_0(m)$, filling the space between us and the cloud, which number is given by researches on the mean distribution in the regular star system. From the sum $A = A_0 + A_1$ the total number $N(m)$ of all stars brighter than m is computed, because the values of N, and not of A, are found directly in practical researches.

In our computations we have made use of the most recent researches on the luminosity curve by Kapteyn's method, made by Dr. Schouten. His results for the number of stars $\phi(\mathrm{M})$ of absolute magnitude M * may be represented by the formula

$$\log \phi = \text{const} - 0{\cdot}029(\mathrm{M}-9)^2.$$

This means that for $\pi = 0''{\cdot}01(\rho = 5)$, $\pi = 0''{\cdot}001(\rho = 10)$, $\pi = 0''{\cdot}0001(\rho = 15)$, the most numerous stars appear as stars of a magnitude $m_0 = 14$, 19, and 24 respectively. Of course the formula has only a real meaning for the values of M between the limits -6 and $+8$, from which it has been deduced. Using these results we deduce

$$\mathrm{A}_1(m) = \mathrm{C} \,.\, 10^{-0{\cdot}029(m-m_0)^2}.$$

For the numbers $\mathrm{A}_0(m)$ we have taken the values found by Van Rhijn (and smoothed by Schouten) for the region of the galactic poles. In Table II. are given the results for log N (the values of A having been computed for integer values of m, the values of N correspond to the halves) and for the gradient $\dfrac{d \log \mathrm{N}}{dm}$.

TABLE II.

m.	$\rho=15$, $m_0=24$. C=2×10⁶. log N.	gr.	$\rho=16$, $m_0=25$. C=5×10⁶. log N.	gr.	$\rho=17$, $m_0=26$. C=12·5×10⁶. log N.	gr.	$\rho=18$, $m_0=27$ C=1/3×10⁸. log N.	gr.	$\rho=19$, $m_0=28$. C=10⁸. log N.	gr.
	8·486		8·486		8·486		8·486		8·486	
6		559		557		556		556		556
	9·045		9·043		9·042		9·042		9·042	
7		530		524		523		522		522
	9·575		9·567		9·565		9·564		9·564	
8		509		494		490		489		489
	0·084		0·061		0·055		0·053		0·053	
9		505		472		458		454		453
	0·589		0·533		0·513		0·507		0·506	
10		532		472		438		425		421
	1·121		1·005		0·951		0·932		0·927	
11		581		496		441		406		392
	1·702		1·511		1·392		1·338		1·319	
12		618		571		488		418		379
	2·320		2·082		1·880		1·756		1·698	
13		624		620		571		487		412
	2·944		2·702		2·451		2·243		2·110	
14		598		629		630		591		517
	3·542		3·331		3·081		2·834		2·627	
15		558		603		637		650		635
	4·100		3·934		3·718		3·484		3·262	
16		509		560		608		649		678
	4·609		4·494		4·326		4·133		3·940	
17		458		511		563		614		662
	5·067		5·005		4·889		4·747		4·602	
18		407		459		512		566		619
	5·474		5·464		5·401		5·313		5·221	
$\mathrm{A}_0=\mathrm{A}_1$	10·3		11·1		12·0		12·8		13·4	
Min.–Max.	8·7–12·7		9·6–13·7		10·6–14·7		11·4–15·6		12·2–16·4	
I	0·207		0·206		0·205		0·218		0·260	

For the bright stars the influence of the star-cloud is nearly (or wholly) zero, but for the higher magnitudes the number of cloud-stars A_1 increases much faster than A_0, and finally makes up the

* W. J. A. Schouten, *On the Determination of the Principal Laws of Statistical Astronomy*, p. 111.

overwhelming majority. The magnitude of transition, for which $A_1 = A_0$, is given at the foot of each table. All the values of log N and of their gradients show the same typical feature: after the normal decrease the gradient reaches a minimum, increases during an interval of about 4 magnitudes—just the interval in which the transition from minority to majority occurs—and then the gradient of log N, now due almost solely to cloud-stars, decreases again. The magnitudes for which the gradient reaches a minimum and a maximum are also given at the foot. They show that for each increase of ρ with unity the reversal of the gradient is displaced over nearly 1 mag. The place of this reversal, therefore, gives a means of finding the distance of the star-cloud.

In one case only the minimum, where the reversal begins, is indicated, as our data do not reach low enough to show the maximum. For the Cygnus patch the minimum lies at $m = 11{\cdot}5$, for the Aquila stream somewhat below the 12th magnitude. These values correspond respectively to $\rho = 18$ and 19. *So we find for the parallax of these parts of the Milky Way* 0″·000025 (*for the bright Cygnus patch*) *and* 0″·000016 (*for the Aquila stream*), *and for their distances* 40,000 *and* 60,000 *parsecs.*

In order to see how far these results depend on our suppositions the computations for $\rho = 18$ and 19 have been repeated, assuming as background A_0 the larger values of the star densities for $l = 20°$ at the border of the galactic zone, and in addition assuming for the density of the star-cloud and for the constant C values twice those first adopted. The results are contained in Table III.

TABLE III.

m.	$\rho=18$, $m_0=27$. A_0 for $l=20°$.		$\rho=19$, $m_0=28$. A_0 for $l=20°$.		$\rho=18$, $m_0=27$. $C=2/3\times10^6$.		$\rho=19$, $m_0=28$. $C=2\times10^6$.	
	9·766		9·766		9·564		9·564	
8	0·265	499	0·265	499	0·054	490	0·053	489
9	0·742	477	0·742	477	0·510	456	0·506	453
10	1·199	457	1·196	454	0·940	430	0·929	423
11	1·640	441	1·631	435	1·365	425	1·328	399
12	2·077	437	2·050	419	1·835	470	1·733	405
13	2·535	458	2·472	422	2·407	572	2·212	479
14	3·043	508	2·928	456	3·073	666	2·823	601
15	3·608	565	3·452	524	3·761	688	3·522	699
16	4·199	591	4·039	587	4·426	665	4·227	705
17	4·782	591	4·650	611	5·045	619	4·899	672
18	5·332	550	5·245	595	5·613	568	5·521	622
$A_0 = A^1$	13·7		14·5		12·2		12·9	
Min.–Max.	11·8–16·5		12·5–17·2		10·7–14·8		11·7–15·7	

Comparing these results with Table II., we see that with a denser background the reversal of the gradient is weakened and displaced towards the fainter magnitudes; by doubling the value of C the opposite effect is obtained, the reversal becoming stronger and steeper, and taking place at a lower value of *m*. In the first case the values of ρ corresponding to the minimum at 11·5 and 12·2 become half a unit smaller, in the second case somewhat less than a unit greater.

The first case is quite probable, as a star-cloud at such a distance is seen through a star-filled space much deeper than our universe reaches in the direction of the galactic poles. But we are not entitled to make arbitrary suppositions as to the density of the cloud or its constant C, for the total light of this cloud is observed as brightness of the galactic light. The brightness, *i.e.* the quantity of light per square degree, expressed by the number of stars of magnitude 1·0 is given by

$$I = \int C \, . \, 10^{-0.029(m-m_0)^2 - 0.4(m-1)} dm,$$

where the integral may be taken between the limits $\pm\infty$ because all the stars are telescopic. Its value is thus

$$I = C\sqrt{\frac{\pi \log e}{0.029}} \, 10^{-0.2(2m_0 - 8.896)}.$$

This I is given at the foot of Table II. Now we do not know exactly the brightness of these patches; the measures of Yntema give 0·088 for the mean brightness of the Milky Way, and Van Rhijn gives 0·057 for $l = 10°$.* So it appears quite possible that a bright part of the Milky Way is 4 or 5 times brighter than this value, thus corresponding to the value of C adopted in Table II.; but it is doubtful whether we may double this value, as we do in the second half of Table III., by doubling the value of C.

4. For the Cygnus patch we have yet another means of finding its distance, by assuming that its light is simply superposed on the same background we are seeing as a faint nebulous light on its eastern side. Then the number of stars of each magnitude belonging to the star-cloud alone is found by subtracting the numbers belonging to the faint Cygnus region from the numbers for the bright patch. We thus have:

m.	N.	Log N.
9·32	11·07 − 10·48 = 0·59	9·77
11·97	182 − 150 = 32	1·50
12·89	426 − 326 = 100	2·00
14·67	5461 − 1522 = 3939	3·60

The number of stars brighter than *m* is proportional to the integral

$$\psi(x) = \int_{-\infty}^{x} 10^{-0.029(m-m_0)^2} dm,$$

where $x = m - m_0$.

* *Groningen Public.*, 22, 34; 27, 55.

The values of log ψ and its gradients are:

$x = -18$	$\log \psi = 0.110$	$\frac{d \log \psi}{dx} = 1.049$
-17	1.159	
-16	2.151	0.992
-15	3.086	$.935$
-14	3.964	$.878$
-13	4.785	$.821$

Between 12.89 and 14.67 the mean gradient is 0.899, corresponding to $x = -14.9$; between 11.97 and 14.67 it is 0.777, corresponding to $x = -12.7$. This gives for m_0 the values $13.8 + 14.9 = 28.7$ and $13.3 + 12.7 = 26.0$, corresponding to $\rho = 19.7$ and 17.0. In this way we find for the distance values of the same order as above, the corresponding values of π being $0''.000013$ and $0''.000040$.

5. The foundation of these determinations of parallax and distance is the luminosity curve deduced by Kapteyn's method. As regards the numerical values deduced from observational data (reaching from $M = -6$ to $+8$) the curve deserves great confidence, and between these limits the formula adopted is fixed by them with great accuracy. Moreover, in computing the reversals of the gradient the formula has only been used as a formula of interpolation; we could have used the numerical values themselves as well, and so only the extreme values for $M = -5$ and -6, $m - m_0 = -14$ and -15, are somewhat uncertain. In computing the brightness, I, however, and also in the second method for the Cygnus patch, the formula is used beyond the limits of observational data. But practically the value of I is also nearly independent of the formula, as the few bright stars with $M < -6$, and the bulk of the very faint stars with $M > 8$, contribute only a small percentage to the value of the integral for I.

The weak point in our method of research lies in the lack of homogeneity in the observational data for N. The star counts of Herschel and Epstein are distributed fairly equally over the regions examined, but the *Carte du Ciel* plates are not, so irregularities in the distribution of the stars may more or less affect the course of the N. Our calculations must be considered as a first attempt to try a method which opens the possibility of studying the internal structure of the galactic system and the location and distribution of its streams and clouds in space as soon as the present scantiness of data is removed.

6. In Part 2 of his "Studies based on the Colours and Magnitudes in Stellar Clusters" Harlow Shapley estimated the diameter of our galactic system at 3000 parsecs, so that from the more remote star cluster Messier 13 it would appear like a nebulous patch of 5° diameter. This view may represent the actual ideas of the dimensions of the galactic system at that time. Afterwards, in his seventh paper, Shapley found that the system of the globular

clusters, which extends in the direction of galactic longitude 325° to a distance as far as 60,000 parsecs, must be connected with the galactic system, because nearer than 1000 parsecs from the galactic plane these clusters are absent. The results we have arrived at here are in accordance with these later views, as they place some bright parts of the Milky Way at a distance of 40–60,000 parsecs. So the starry masses of the Galaxy are spread over space as far as the remotest clusters, and clearly both belong together to one system. In this system the dense agglomerations of stars are spread over a flat disc of about 2000 parsecs thickness, and in the empty space above and below it the globular clusters are dispersed. Shapley explained the absence of these clusters from the galactic disc by the absorption of the dark nebulous masses filling this space. It may perhaps also be explained by assuming that globular clusters, when coming from the void space into the star-filled galactic regions, are gradually broken up and dispersed into open clusters by the attraction of these stars.

Now, Shapley's result, that in the universe of globular clusters the sun occupies a very eccentric position, is contrary to the common view, which places the sun in our galactic system not far from the centre. Therefore, it must be emphasised that Shapley's result is wholly in accordance with the aspect of the Milky Way. For the hemisphere with centre at $\lambda = 325°$, where judging by the abundance of globular clusters our universe reaches farthest, we observe in the direction of Sagittarius a series of round or irregular and rather small bright patches and clouds, which probably lie at a great distance. At the opposite side of the heavens we have at $\lambda = 100°$ to $140°$ the faint region of Perseus, where the galactic light nearly vanishes; on both sides of it the light band gradually becomes brighter. So it is very probable that the galactic star masses on the side of Scorpio and Sagittarius reach many times farther than on the opposite side. The sun must then be situated near to the limit of the system in the direction of Perseus.

Note on the Blue-Violet Absorption of Venus.

Arthur W. Clayden, M.A.

In Mr. Evershed's interesting report on the work of the Kodaikánal and Madras Observatories, published in the February number of the *Monthly Notices*, a passage occurs which suggests some comment. It runs thus:—

"In photographing the spectrum of Venus with the grating spectrograph in the blue and violet regions, it was noticed that longer exposures were required than is necessary when the image of a brightly-illuminated cloud is brought on to the slit. Direct comparisons of the spectra in a low-dispersion prism spectrograph,