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$$(v-b)\,b'' = -\frac{b'}{m^2}\left[\frac{x+2\varphi}{x+\varphi} + \frac{x+\varphi}{x+1}(1+x\beta)(2\beta-1) + x\frac{mb'}{\varphi}\right].$$

Replacing in this mb' by $\frac{1}{x+1}\beta(1-\beta)\varphi(x+\varphi)$ (see above), we get:

$$(v-b)\,b'' = -\frac{b'}{m^2}\left[\frac{x+2\varphi}{x+\varphi} + \frac{x+\varphi}{x+1}\left\{(1+x\beta)(2\beta-1) + x\beta(1-\beta)\right\}\right],$$

i e.

$$(v-b)\,b'' = -\frac{b'}{m^2}\left[\frac{x+2\varphi}{x+\varphi} + \frac{x+\varphi}{x+1}(x\beta^2+2\beta-1)\right], \quad \ldots \quad (2)$$

identical with (12) in I, p. 293, paying attention to the above expression (1) for b'. This derivation seems somewhat shorter to me than that in I, p. 292—293, and besides confirms the validity of the result, so that we can calculate b''' from it with full assurance.

Clarens, Nov. 13, 1911.

(*To be concluded*).

Astronomy. — *"A photographical method of research into the structure of the galaxy."* By Dr. A. PANNEKOEK. (Communicated by E. F. VAN DE SANDE BAKHUYZEN).

(Communicated in the meeting of November 25, 1911).

In my paper: "Researches into the structure of the galaxy", published in the Proceedings of the Meeting of June 25, 1910, I have pointed out that the chief difficulty in this kind of researches consists in the lack of completeness and homogeneity in the material of star-countings that is at our disposal. HERSCHEL's and EPSTEIN's gauges and the countings on photographic and other stellar charts have relation to small, generally non-coincident parts of the examined galactic region. Owing to this the fluctuations of density, which may be considerable even in smaller regions (comp. MAX WOLF's and BARNARD's photographs of the Milky Way), appear with their full amount as errors of the function $N(m)$ (number of stars per square degree as function of the limiting magnitude m). At best we may only hope that in the mean of a great amount of countings these irregularities lose their influence. Still there always remains uncertainty and doubt justifying the question whether these drawbacks may not be avoided by another method.

This may be done by having recourse to photography. In this manner the chief condition may be fulfilled at once : for all magnitudes m exactly the same part of the heavens is taken and counted. *Hence the local differences in star-density become altogether harmless*; as the irregularities shown by the function $N(m)$ itself are probably much smaller, this function may now be more easily deduced. If all numbers N for different m can be determined *on one and the same exposure* then the influence of the transparency of the air, of the different sensitiveness of the plates, of the development etc., disappears at the same time.

Another condition which is of the greatest importance for the practicability of the method, consists in the fact that for the determination of N *only countings* are wanted without estimates or measurement of the brightness of the stars.

One can e.g. make two photographs on one plate with the same time of exposure, one without and one with a screen interposed, the absorption of which is known to be a magnitudes. This absorption may be determined by photometrical measurements, most precisely by measuring the diminution of surface-brightness by the screen. This is equal to half of the stellar absorption in magnitudes and it is independent of the wave-length of the light. By counting the stars appearing only on the photograph taken without the screen and the stars showing both images we get two numbers $N(m)$ and $N(m-a)$, for which we accurately know the difference a of the limiting magnitudes, which is specially important for determining the value of the gradient $\frac{d \log N}{dm}$. The absolute value of m may be found by measuring a few individual stars. We might also in order to find a third value, count the number of stars, for which moreover the first diffraction-image is visible. But only when the gauze is so coarse that the first spectrum is contracted practically into a star-shaped image, its visibility depends on the entire quantity of light. Otherwise the limiting visibility depends also on the colour of the stars and the difference in visibility between the diffracted and the principal image is theoretically unknown.

Another method will be more practical. We take a number of photographs on the same plate with *geometrically progressing times of exposure*. Of each star we then obtain a series of images, each time differing in brightness a constant number of magnitudes. Now we simply have to count the number of stars with one visible image, with two images, with three, with four, with five images. These give the numbers N corresponding with the limiting magnitudes m, which increase each time with a constant amount. To remove the uncertainty owing to the confusion of the faintest star-images with spots in the plate it is advisable to take the photograph of longest exposure twice. The faintest stars are then all double and undoubtedly recognisable, while there can be no doubt about the visibility of the faintest image of the brighter stars, because the place where we have to look for it is accurately known.

In this method, however, the difference in brightness between the several images is not known beforehand and must therefore be determined by special measurements. We know approximately how many magnitudes are gained by a certain lengthening of the time of exposure, [1] but this increase differs for different plates and must therefore be determined for each plate by itself. Therefore a scale of photographic magnitudes must be fixed on the plate. These magnitudes cannot be derived from a scale of visual brightness since the spectra of the faint stars are unknown. It must be performed independently by photographic methods.

Up till now there does not exist a scale of photographic stellar magnitudes (defined by $m = 2.5 \log L$), which is independent of visual brightness. Generally it is determined in such a manner that they correspond for the stars of a certain spectral class (HARVARD A.) to the visual photometric scale. Also in PICKERING's "Report on stellar magnitudes in KAPTEYN's selected areas" [2]) the necessity for fixing the scale of magnitudes for the fainter stars by means of prismatic companions, of exposures of different length or with wire-gauzes, is only mentioned in general terms. It is plain, however, that the first two methods cannot give an independent scale-value. This can be done only by a wire-gauze the absorption of which has been determined by physical experiments. Another independent method has been proposed recently by HERTZSPRUNG, viz. to use the first diffracted image obtained with a coarse screen in front of the objective, the strips of which are of the same width as the spaces between them.

So a "scale-plate" must be taken of the same part of the heavens of which a "counting-plate" has been made, in order to determine the magnitudes; on the first one two exposures of the same

[1]) According to SCHWARZSCHILD (Beiträge zur photographischen Photometrie der Gestirne) is $\log I + p \log t = \text{const.}$; the values found for p were generally lying between 0,7 and 0,9. Accepting 0,8 as mean, then with a tenfold time of exposure $\log I = 0{,}8$, and the gain is exactly two magnitudes.

[2]) J. C. KAPTEYN, First and second Report on the progress of the plan of selected areas 1911. p. 31.

duration have to be made, one with a free objective, and one with a wire-gauze, or according to HERTZSPRUNGS's method, the second one with the coarse screen. We thus obtain two images of each star on the plate, differing a *known number of magnitudes* and from this there may be deduced a scale of photographic magnitudes on the plate, which is altogether independent of visual magnitudes.

This method, however, has this drawback that the two exposures are taken one after the other, so that, owing to changes in the transparency of the atmosphere the real difference in brightness does not correspond to the difference found on the plate. Hence the basis of the method becomes doubtful. In order to escape this objection another photograph should be taken at the same time with a second apparatus to control the first. Another means is the application of SCHWARZSCHILD's method in which a fine wire gauze is interposed in the cone of rays a little way before the focus. This produces the same kind of images as a gauze before the objective and the diminution of brightness of the principal image is the same. If for the first exposure the gauze is placed before one half of the plate, leaving the rest uncovered, and if for the second the gauze is placed before the other half of the plate, we obtain two images of every star on the plate, one undiminished, the other weakened, except in the central zone which is no use because the rays partially passed through the gauze. If a stands for the absorption-coefficient, d for the difference in brightness owing to the change of the atmospherical transparency, then the difference of the images on the one half of the plate is $a + d$, on the other half $a - d$, and in the mean the influence of d disappears.

As soon as an accurate, independent and reliable scale of photographic magnitudes has been fixed for a definite region of the heavens (e. g. for the zone near the north-pole), then we can determine the stellar magnitudes of any other part simply by taking it together with the polar region on one and the same plate. As long as this ideal has not been attained the scale must be fixed for each region individually, for which purpose SCHWARZSCHILD's method with a half gauze in the cone of rays seems the most practicable.

For a practical determination of the stellar magnitudes we can measure the diameters of both the undiminished and the weakened images of a number of stars forming a series of decreasing brightness. For the faintest images a scale of blackness gives a continuation of the scale of diameters. From this the stellar magnitude as function of the diameter may easily be computed. The same end, however, may be attained just about as accurately by estimates without the need of measurements. To this end a scale of star-images must be construed, regularly progressing through properly chosen times of exposure with $^1/_5$ or $^1/_4$ magnitude. If a group of stars is photographed for instance with exposures of 9, 12, 16, 21, 28, 38, 51, 67, 90, 120, 160 213, 285, 379, 506, 675 and 900 seconds, we obtain a scale covering 4 magnitudes with intervals of 0,25, between which each stellar image on a plate may be interpolated. It does not matter whether the difference between the extremes deviates more or less from 4 magnitudes. This scale is only a makeshift and fulfils as it were the part of a scale of millimeters in which each star is classed by measuring its diameter. With the help of a few of such overlapping scales, covering together a still greater range of stellar images it is possible to determine by estimate both the images of each star on the scale plate. The known difference between the two images enables us to reduce the brightness expressed on this provisory scale (just as the millimeter-scale of the diameters) into real magnitudes. Thanks to this the photographic brightness of a number of stars, from the brightest to the faintest, is known. The absolute value, the zero-point, may be deduced from a few bright stars of from the 6th to 8th magnitude, and need only be roughly known.

When in this manner the magnitudes of a number of stars have been determined on the scale-plate, which has to be exposed a little longer than the longest exposure of the counting plate, there only rests to be seen which of these stars show 1, 2, 3, 4, 5 images on the counting-plate. In this way the limiting brightness of each of these classes is immediately given.

With this method we therefore only want — besides the scale for estimating the magnitudes — 2 plates for each region of the Milky Way: a counting plate and a scale-plate. Practically without any measurements, hence without any other instrument than a magnifying-glass, only by countings and estimates we can thus obtain all the data which must otherwise be compiled with a great deal of trouble and far more incompletely from the different sources of catalogues, stellar charts and gauges. As Professor SCHWARZSCHILD, Director of the Astrophysical Observatory at Potsdam graciously offered to have a few plates made for this purpose with the exceedingly appropriate Zeiss-triplet of 1500 mm. focal-distance and 150 mm. aperture, and as Mr. HERTZSPRUNG astronomer at the same observatory, kindly made the photographs for me, I take these as a standard of what may be attained in this direction.

On a photograph of the polar region with this instrument on an exceedingly sensitive Lumière Sigma-plate a 10 minutes' exposure showed stars up to magnitude 13,0 according to PICKERING's scale. Consequently when we use on the counting-plates exposures of 1900, 600, 190, 60, 19, 6 seconds, the limiting magnitudes of the different classes of stars are 14,0 13,0 12,0 11,0 10,0 9,0. So they penetrate just as far into the region of the faintest stars as HERSCHEL's gauges do. The field within the limits of which the stars retain their perfectly round and well-defined shape has a diameter of 6°; so one plate can be used to cover a part of the heavens of more than 25 square degrees.

Here lies before us a field for research on which with little trouble and simple implements very important data can be gathered for the structure of the galactic system. The necessary photographs take but little time; the deduction of the results wants more, but on the other hand asks very little instrumental help; hence this is a very suitable field for the cooperation of observatories with persons who cannot command large instruments. Formerly it often happened that amateurs gathered valuable data by the observation of the heavens. This will gradually have to cede its place in future to the discussion of stellar photographs as an opportunity to cooperate succesfully to the promotion of science.

Mathematics. — *"Homogeneous linear differential equations of order two with given relation between two particular integrals"*. By Dr. M. J. VAN UVEN. (Communicated by Prof. W. KAPTEYN). (2^{nd} communication).

(Communicated in the meeting of November 25, 1911).

In the preceding communication we made the two rectangular coordinates x, y of a point satisfy the same homogeneous linear differential equation of order two:

$$\frac{d^2x}{dt^2} + p(t) \cdot \frac{dx}{dt} + q(t) \cdot x = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

and showed a given relation between x and y makes also a functional relation between p, q and t. This was evident particularly when we brought the equation (A) in a standard form, either in the form

$$\frac{d^2x}{d\tau^2} + \frac{I(\tau)}{2} \cdot \frac{dx}{d\tau} + x = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

or in the canonical form: