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The Planetary Theory of Ptolemy

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I

In Ptolemy's "Mathematical Composition" ancient astronomy found its most perfect and complete exposition. The planetary motions, occupying the second half of this treatise, are explained by means of the theory of the epicycles, the ripest product of ancient science. Its fundamental ideas had been developed by Apollonius and Hipparchus; but it was Ptolemy who worked it out completely.

In representing the motions of the planets the epicycle theory may be said to be entirely equivalent to the later heliocentric theory. If we assume the earth to be at rest and give to each of the celestial bodies its motion in the opposite direction, the sun thus describing a yearly orbit around the earth and the planets getting a second circle besides their own orbits, then the relative motions remain unchanged and the phenomena are exactly the same.

In Greek astronomy all heavenly bodies move in circles. A planet moves on a circle (epicycle) the centre of which describes a greater leading circle (deferent) around the earth as the world centre. For the outer planets (Saturn, Jupiter, Mars) the leading circle is their own orbit, the epicycle is the reflected orbit of the earth. For the inner planets (Venus, Mercury) the leading circle is the reflected earthly orbit, the smaller epicycle is the planet's orbit.

How will the elliptic motion of the planets appear in this world system? The deviation from a circular figure is, except for Mercury, too small to be perceived. But the eccentricity and the variable velocity will show themselves exactly as they would appear if seen from the sun. Since the velocity of the planet is greatest at perihelion, where moreover it is seen at a smaller distance, the effect of the eccentricity on the apparent, *i.e.* the angular, motion is doubled; so it produces an inequality in longitude, an alternative anticipating and lagging relative to a regularly progressing "mean" planet, of double the amount of the real eccentricity. At the same time the epicycle, because, in moving around, it is seen at different distances from the central body, varies in apparent size proportional to the eccentricity itself; the oscillations producing the direct and retrograde motions thus are variable to an amount of the single eccentricity.

Hipparchus had already explained the variable angular velocity of the sun in its yearly course along the ecliptic by means of the eccentric position of the earth. Since he assumed a constant velocity of the sun the eccentricity had to be double the amount of its real value (0.017); Hipparchus took it to be 1/24, and Ptolemy assumed the same value.

Hipparchus had not worked out a complete epicycle theory of the planets, probably from lack of sufficient observational data. This was left to Ptolemy. His pride in having achieved it is evident in the opening words of the parts devoted to the planets:

"It now being our purpose to prove also for the five wandering stars, just as for the sun and the moon, that their seeming anomalies are all of them fulfilled by means of uniform and circular motions, because these motions conform to the nature of divine beings but disorder and dissimilarity are foreign to them, the happy success of such an enterprise may be considered a great thing, nay in reality the final goal of philosophically directed mathematical science; but for many reasons it is difficult and therefore naturally has not been undertaken successfully by anyone as yet." (Book IX, Chapter 2).

Ptolemy was an active observer; clearly he made many more observations than the small number selected among them which he quotes and uses in his book. It is said that his observations are less precise than those of Hipparchus; the positions of the stars are given by him, according to Dreyer, to ¼°, by Hipparchus to ⅙°. The errors of the planets' positions, however, are much larger. Ptolemy says that he used for his demonstrations only such observations as offered the best warrant of reliability: either conjunctions with, or near approaches to, fixed stars or the moon; or measurements with the astrolabe. This instrument, composed of graduated rings corresponding to the celestial circles, was first turned into the true position by means of a star, or the sun or the moon; then the planet was observed through diopters and the circles were read. Besides his own observations between 130 and 142 A.D. he made, for earlier years, use of observations by Theon at Alexandria. The periods of revolution which he took from Hipparchus, probably came from the Chaldeans; in some cases he used slightly corrected values.

II

Theory is most simple and in conformity to our ideas for the outer planets. The planet travels along the epicycle with constant velocity in such a way that it keeps in pace with the sun in its orbit, *i.e.* that the radius from the epicycle-centre C to the planet P remains always parallel to the radius from the centre of the sun's orbit to the sun. The arc it performs is counted from the outermost point of the epicycle F, farthest removed from the centre of the deferent; the ancients considered that outer point as being at rest when the epicycle was carried around, just as if, like a wheel, it were fixed at the end of a turning rod. That arc is called "anomaly relative to the sun"; Figure 1, where it is represented by *F P*, shows that it is equal to the difference between the longitude of the sun (direction *ES* or *CP*) and the longitude of the epicycle's centre (direction *EC*), called "mean longitude of the planet." When the arc completes 360° the planet returns to the outer point of the epicycle, it stands again in conjunction with the sun, and has performed a (synodic) planetary period. The deferent is an eccentric circle, with the earth *E* somewhat outside the centre *M*; it is usually called the "excenter" by Ptolemy. The arc *VC* on this excenter performed by the epicycle's centre, counted from the vernal point *V*, is the "longitude relative to the ecliptic"; if it reaches 360° a revolution of the planet is completed. Such is the original form of the epicycle theory. But Ptolemy perceived that it was not sufficient.

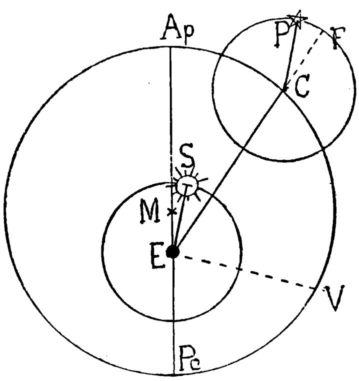

FIGURE 1

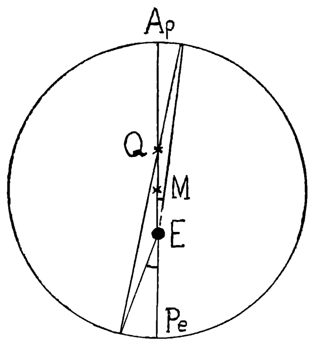

FIGURE 2

"Now we found, however," he says, "by our continued testing and comparison of the successively observed courses with what by reasoning resulted from the combination of the two hypotheses, that the course of motion cannot be so simple." First the directions of apogee and perigee (points of the deferent most and least remote from the earth, *Ap* and *Pe* in Figure 2) partake in the precession of the stars. "We found that first the apogees of the excenters make a small progress in the direction of the signs, uniform again around the centre of the ecliptic and for all the planets nearly the same as has been observed for the sphere of the fixed stars (*i.e.* one degree in one hundred years), in so far at least as it is possible to form an idea from the available data. Secondly we found that the centres of the epicycles go round on circles of the same size as the excenters causing the anomaly, but having another centre: for the other planets the centres of these circles are situated midway between the centres of the above-named excenters and the centre of the ecliptic." (Book IX, Chapter 5). Expressed in other words: the point around which the epicycle's centre revolves with constant angular velocity—afterwards called *punctum aequans*, equality-point, equant, *Q* in Figure 2, is not the centre of the circle described but is situated as far outside this centre M to one side as the earth (the "centre of the ecliptic") is distant toward the opposite side.

This is indeed an excellent representation of what at the time could be observed of the planetary motions. The distance of the epicycle's centre to the earth, hence the apparent size of the epicycle, varies according to the eccentricity ME. That the arcs described in equal times appear as equal angles as seen from the equant means that in reality the velocity at apogee is smaller, at perigee is larger; seen from the earth at the other side of the centre the difference in angular velocity shows the double effect of the eccentricity. Thus the distance earth-to-equant, often called the total eccentricity, determines the variation in angular velocity, whereas the distance earth-to-centre of the circle is what we call the real eccentricity. In the motion of the sun only the former presents itself.

Ptolemy does not give any observations to demonstrate this theory. He could have done so easily; observations of Mars far away from opposition, once in the vicinity of apogee, another time in the vicinity of perigee, would clearly show the differences in apparent size of the epicycle. Nothing of the sort is found; he simply states the theory. Of course he must have found it from observations. In his introduction he says:

"When we are compelled to premise certain axioms not deduced from a visible datum but found by coherent trying and inferring . . . we do it with the knowledge . . . that axioms given without demonstration, if they prove to be concordant with observation, cannot have been found without methodical argument though it is difficult to explain how they have been derived . . ." (Book IX, Chapter 2). We may be sure that observations have shown to him the working eccentricity in one case nearly half as large as in the other case, and that the assumption of an exact ratio 1:2 was a theoretical but lucky hit. He presents the result in this way: "Also for these planets the eccentricity, deduced from the greatest deviation in the anomaly relative to the ecliptic, was found to be the double amount of the eccentricity derived from the retrograde motion in the cases of the largest and smallest distance of the epicycle." (Book X, Chapter 6).

Ptolemy's first task is now to ascertain, for the planets Mars, Jupiter, Saturn, the elements of their excenter, *i.e.*, the eccentricity and the direction of the line of apsides, expressed by the longitude of the apogee. In this he makes use of three oppositions, the moments when the planet is at the deepest point of the epicycle, nearest to the earth, because at such a time the observed longitude of the planet coincides with the longitude of the epicycle's centre. We call it opposition, but it is not opposition to the real sun but to the "mean" sun; it is not the line earth-sun but the line centre of solar circle-sun that turns regularly in pace with the planet on its epicycle; in this Figure 1 was not entirely exact. By comparing for every day the longitude of the mean sun, taken from his tables, with the observed longitude of the then retrograding planet, the moment and the longitude of opposition could easily be derived. "Thus first we ascertained for Mars three oppositions. We observed it in Hadrian year 15, at 26/27 of the month Tybi (transformed into our calendar December 15/16 of 130 A.D.) 1^h in the night at Gemini 21° (*i.e.*, longitude 81°); in Hadrian year 19, at 6/7 Pharmuthi (February 21/22 of 135 A.D.) 9^h in the evening at Leo 28° 50′, (*i.e.*, longitude 148° 50′); in Antonin year 2 at 12/13 Epiphi (May 27/28 of 139 A.D.) 2 hours before midnight at Sagittarius 2° 34′ (i.e., longitude 242° 34′)." (Book X, Chapter 7.)

Now the problem is a purely geometrical one. Asked is the position of the earth E within the eccentric circle. Given are the longitudes of the planet at I, II, III (Figure 3), hence the angles 67° 50′ and 93° 44′

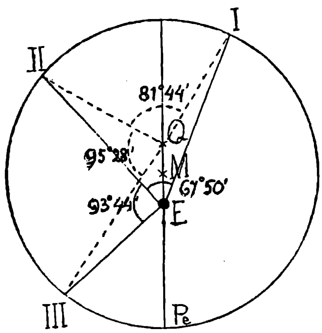

FIGURE 3

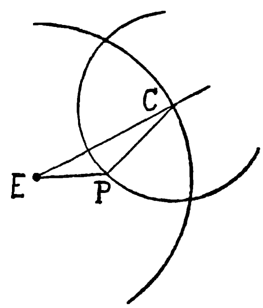

FIGURE 4

at E. Moreover the time intervals, diminished by a certain number of full revolutions, determine the proportional angles at the *punctum aequans* Q: 81° 44′ and 95° 28′. Ptolemy remarks that this problem is too difficult to be solved in a direct way. Hence he solves it by a series of approximations. He first supposes Q to be the centre of the circle, and finds distance and direction of EQ. Then he computes what difference in the three observed longitudes the transition to a circle with centre midway between E and Q would produce; after a new computation with the corrected values the process is repeated once more. In this simplified form Ptolemy's problem was identical with what in modern geodetics is called Pothenot's or Snell's problem: to determine a position between three well-known points by measuring at the unknown station the directions to these three points. His solution is straight-forward, of course somewhat more clumsy than it is with our modern trigonometrical formulas, because he had at his disposal only his table of chords for every angle (the ancient form for what we know as a table of sines) and Pythagoras' theorem, with now and then an appeal to some proposition of Euclid. The results were:

First computation $EQ = 0.219$, arc III $- Pe = 39° 19'$.
Corrections to I, II, III $-0° 32'$, $+0° 33'$, $-0° 50'$.
Second computation $EQ = 0.197$, arc III $- Pe = 45° 33'$.
Correction to I, II, III $-0° 28'$, $+0° 28'$, $-0° 40'$.
Third computation $EQ = 0.200$, arc III $- Pe = 44° 21'$.

From the latter value the angle III *EPe* is computed to be 52° 56′, so the apogee is situated at Cancer 25° 30′, *i.e.*, at longitude 115° 30′.

His second task was to derive the size of the epicycle relative to the excenter. It can be done most easily by using an observation of the planet outside its opposition, when it stands far sideways on its epicycle. For this derivation Ptolemy, most curiously, gives an observation only 3 days after opposition. "Since it is our further aim to fix numerically the relative size of the epicycle, we chose an observation we made nearly 3 days after the third opposition, in Antonin year 2 at 15/16 Epiphi (May 30/31 of 139 A.D.) 3^h before midnight." With the astrolabe directed by means of Spica (longitude 176° 40′) the longitude of Mars was found to be 241° 36′. Mars at the same time was found to stand 1° 36′ east of the moon (which according to the tables stood at 239° 20′, corrected 40′ for parallax giving 240° 0′). Hence also in this way of reckoning "a position of Mars was found 241° 36′, agreeing with the other result." (Book X, Chapter 8.) The exact concordance of the minutes of arc clearly indicates that they are fitted on purpose. If we try to verify the computation we find the longitude of the moon from the tables 13′ larger; since, however, the time is only given in hours and the moon moves 33′ per hour, every value within this range might be taken; the choice has been made in such a way that the concordance was absolute.

With these data it is again simply a computation by geometry. The method followed comes down to this: Mars as seen from the earth had retrograded 0° 58′ since the opposition; the centre of the epicycle was computed to have proceeded 1° 45′, so that relative to this centre the planet has regressed 2° 43′, whereas on the epicycle an arc of 1° 8′ had been described. From the triangle *EPC* (Figure 4) with $\angle E = 2° 43'$ and $\angle C = 1° 8'$ and the distance *EC* at the time 0.934 we find *CP*, the radius of the epicycle to be 0.658. The true value is 0.656. It is clear that this close accordance is either chance luck or design. If errors of 15′ are allowed in both observations, at opposition and 3 days afterwards (the real errors may be considerably larger) then the result may be 0.030 different. So we may take it for certain that his value for the size of the epicycle does not rest simply on this observation, but has been derived from other and more favorable observations with angles of 20° instead of 2°. Then the derivation given in his book has to be considered as a demonstration of the geometric method followed; in that case it may be understood that the figures of the minutes are adapted and chosen in such a way as to give the good result. It must be added that computed with modern data the longitude of Mars at the moment of his observation was 242° 16′, hence 0° 41′ larger. It is well known that owing to an error of his vernal point of about 1° all longitudes of Ptolemy, including those of the solar tables, are too small; for Spica the error in his catalogue was 1° 19′, so that in the measurement of Mars an error of 39′ was made.

In the same way as is explained here for Mars Ptolemy derived the elements for Jupiter and Saturn. His results are contained in the following small table, where for the sake of comparison the true values according to modern data are added.

TABLE 1

ELEMENTS OF THE OUTER PLANETS

	Mars		Jupiter		Saturn	
	Ptolemy	True	Ptolemy	True	Ptolemy	True
Total eccentricity	0.200	0.186	0.092	0.096	0.114	0.112
Long. of apogee	115°.5	121°	161°	164°	233°	239°
Radius of epicycle	0.658	0.656	0.192	0.192	0.108	0.105

The smallness of the differences shows in how excellent a way ancient astronomy, in Ptolemy's work, succeeded in representing the phenomena of these planets.

III

For the inner planets the problem is more difficult, because the premises of the theory fit less well the phenomena. Here the epicycle is our real orbit of the planet and is supposed to be a circle described with constant velocity. The deferent represents our orbit of the earth, hence should be identical with Ptolemy's solar orbit. His theory, however, only demands that the radius in both have always the same direction, *i.e.,* that the mean longitude of the planet is equal to the longitude of the mean sun. The real sun is entirely disregarded; the identity of the mean motions secures that always Mercury and Venus go on oscillating from one side of the sun to the other. Ptolemy in his theory deals only, as data of observation, with the greatest elongations east and west; he has to represent them by the motion solely of the epicycle's centre along the deferent.

For Venus matters are facilitated by its small eccentricity, in modern theory 0.007 only with perihelion at 105°; the greatest elongations vary between 45° 53′ and 46° 43′ only. This is relative to the true sun which on account of the eccentricity of the earth's orbit (0.017 with perihelion at 70°) at longitude 340° is 1° 57′ ahead, at longitude 160° is 1° 57′ back relative to the mean sun. At a solar longitude of 15° (and 195°) the orbit of Venus is seen sideways, with the centre 0° 22′ behind (or before) the sun. Thus Venus' advancing and retarding relative to the mean sun (*i.e.,* to mean Venus as well) is a combination of both, chiefly determined by the solar eccentricity, slightly modified by that of Venus, amounting to 1° 40′ ahead at longitude 333°. In such a way, with eccentricity 0.029 and apogee at 63° the phenomena can be rendered by such a theory as Ptolemy's.

Ptolemy makes use of 8 greatest elongations of Venus observed by himself (5) or by Theon (3); we have collected his data in Table 2. He first looks for the direction of apogee and perigee, for which he takes two greatest elongations of the same amount, to the western and the eastern side; since they can only be equal at equal distance at both sides of the line of apsides, this line must fall midway between the observed longitudes. This is the case with the first pair; its mid-value is 55°; it is also the case with the second pair, with mid-value 235°. Which of them is the longitude of apogee? The third pair gives the answer; with the mean sun near 55° the greatest elongation is smaller, near 235° it is larger; hence the former is the apogee. The fourth pair, with the mean sun at a 90° different longitude, shows the maximum amount of the inequality of the greatest elongations; half the difference, 2° 22½′ corresponds to an eccentricity of the deferent of 0.041 or 1/24.

TABLE 2

GREATEST ELONGATIONS OF VENUS

		Data of Ptolemy			Computed Values		
Date of greatest elong.	Time	Long. ♀	Mean long. ☉	Elongation rel. to mean ☉	Long. ♀	Elongation rel. to mean ☉	Date of greatest elong.
132 March 7	7^h ev.	31° 30′	344° 15′	+47° 15′	31° 13′	+46° 57′	Feb. 21
140 July 30	4½^h mo.	78 30	125 45	—47 15	80 29	—46 9	July 7
127 Oct. 11	6½^h mo.	150 20	197 52	—47 32	150 35	—47 14	Sept. 23
136 Dec. 25	6½^h ev.	319 36	272 4	+47 32	319 57	+46 53	Dec. 4
129 May 20	5^h mo.	10 36	55 24	—44 48	11 58	—44 23	Apr. 30
136 Nov. 18	5^h ev.	282 50	235 30	+47 20	282 4	+45 32	Dec. 4
134 Feb. 18	6^h mo.	281 55	325 30	—43 35	281 58	—44 31	Feb. 17
140 Feb. 18	5½^h ev.	13 50	325 30	+48 20	14 30	+47 59	Feb. 18

What could be expected, *viz.*, that the deferent of Venus shows, somewhat modified, the eccentricity of the earth's orbit, comes out indeed; however, not our true but Ptolemy's too large value, because it entered by means of his solar tables. The curious fact is that Ptolemy himself seems not to have perceived, at least does not mention, the exact coincidence of this result 1/24 with the eccentricity of the sun's orbit, or the near coincidence of the apogee. To him the orbit of the sun and the deferent of Venus clearly are entirely unrelated things that only have the same mean longitude and mean motion.

The third pair of elongations, at apogee and perigee, allows the derivation of some numerical values. The radius of the epicycle, in unit distance of its centre to the earth, is given by the sine of the greatest elongation; for apogee and perigee we find sin 44° 48′ = 0.7046 and sin 47° 20′ = 0.7353. Their mean 0.720 is the semi-diameter of the epicycle; half the difference, percentual, 0.0153 : 0.720 = 1/47 is the eccentricity of the observing point, the earth. It is just half the amount of the total eccentricity found above; hence the Venus-deferent has an equant. Since Ptolemy, obviously, was not aware of the identity of this deferent with the sun's circle, Kepler had afterwards to discover the existence of this equant as a new fact. So it is not exactly true what we said above that he nowhere gives a demonstration of the bisection of the eccentricity; here for Venus he derives the total as well as the simple eccentricity. The exactness of the ratio 1:2 again is suspicious; a change of only 15′ in one of the greatest elongations, smaller than errors which are to be expected, would change the result one-fifth of its value.

There is more in the figures to be wondered at. How did he manage to have the pairs of greatest elongation so exactly equal even to the single minutes, whereas his instruments could only be read to ¼ or ⅙ of a degree? And the results of estimates from conjunctions and near appulses often were more erroneous. By computing the elongations according to modern data—using P. V. Neugebauer's "Tafeln zur astronomischen Chronologie"—we found the values in the right hand columns of Table 2, showing errors in Ptolemy's data of sometimes more than a degree. And how did he manage to have greatest elongations just at moments when the sun's mean longitude had the desired value of 55° or 235°? These eight phenomena of greatest elongation in reality took place at fixed, determinate days not to be influenced by his needs, and he had nothing out of which to choose. If, then, we compute the real dates of greatest elongation (right hand part of Table 2), it appears that his dates deviate, sometimes up to 20 days. It is, indeed, hardly possible to determine the exact moment of observation, as the change is then nearly imperceptible; 20 days before and after the elongation has decreased only 1° 9′ and 1° 54′. So, if Ptolemy says: "We observed Venus on Dec. 25, 136 A.D., in greatest elongation from the sun," he understands under that name a longer period, more than a month, during which no change of elongation is perceptible. This is shown quite clearly where, shortly after having used this elongation of Dec. 25 he says: "We observed 136 A.D. on Nov. 18 Venus in greatest elongation." This represents the same greatest elongation, which in reality took place on Dec. 4. Out of such an extended period of greatest elongation he took such observations as satisfied his needs as to the longitude in the orbit; if they did not exactly fit he could interpolate the result for predeterminate conditions. Here again we get the impression that for working out his theory more observations have been used, and that in his book he has chosen such as were required for an orderly geometric demonstration.

IV

Mercury presented far greater difficulties; indeed its large eccentricity (0.21 with perihelion at 48°) makes its motion highly unfit to be represented by the epicycle theory. The greatest elongation from the true sun varies between 27°.8 and 17°.9; the centre of its ellipse that has to follow the deferent oscillates to both sides of the sun to an amount of 4°.7 sin (L — 48°) (for L = 138° when the ellipse is seen sideways the centre is 0.21 × 0.387 = 0.0812 = 4°.7 ahead of the sun). The true sun deviates, as we have seen, an amount of 2°.0 sin (L —250°) from the mean sun; adding the two oscillations we find that in total the centre of Mercury's epicycle advances or follows its mean position by 2°.8 sin (L —32°). An excenter with total eccentricity 0.049 and apogee at 212° can represent it.

The apparent size of the epicycle, however, visible in the sum total of east and west elongations, in this case does not vary only by virtue of the eccentric orbit of the earth which makes it deviate from its mean value 22° 50′ by 1/60 in surplus at 70°, by 1/60 smaller at 250°, but also by the elliptic shape of Mercury's orbit. When looking in the direction of the line of apsides (at 48° and 228°) we see the transverse axis to be 1/45 smaller than the great axis. Combining these changes we find that the greatest elongation at longitudes of about 50° to 70° is 1/60 + 1/45 = 1/26 smaller (nearly 1°), at 140° — 160° and 320° — 340°, nearly normal; and at about 230° — 250°, 1/45 — 1/60 = 1/180 smaller, hence practically normal also. To represent these variations the deferent should exhibit in three quadrants nearly the same distance to the earth, but in the fourth quadrant (about 50° — 70°) a distance 1/26 larger. This is not only inconsistent with the excenter just derived but cannot even be represented by a circle, being rather egg-shaped.

Now in Ptolemy's theory we find just such an orbit, produced by an effectual geometric construction. The centre of Mercury's deferent is not a fixed point but describes yearly in the opposite direction a small circle with radius 0.05 and centre at a distance 0.10 from the earth. In drawing the figure we see that the orbit now is changed into an oval extended in the line of apsides, flattened in transverse direction. The distance to the earth in perigee is 0.95, in apogee 1.15, but at 60° to both sides of perigee it is smallest, 0.93; hence only in one quadrant it exceeds perceptibly the value in other directions. At first sight this seems to be a remarkable concordance; but it disappears when we see that the apogee is situated at 190°, not far from that of our first-mentioned excenter, and the excess in distance is 1/5 instead of 1/26. So his complicated non-circular orbit cannot have any reality.

He derived it, at least demonstrates it, in the same way as for Venus. He makes use of 4 pairs of observed greatest elongations, which are collected in our Table 3. The first two pairs fix the line of apsides in the midst of the two solar longitudes, at 10° and 190°; the third pair decides that 190° is the apogee. Moreover this pair of elongations shows the distances in apogee and perigee to have the ratio 23 :19, conforming

TABLE 3

Greatest Elongations of Mercury

		Data of Ptolemy			Computed Values		
Date of greatest elong.	Time	Long. ☿	Mean long. ⊙	Elongation rel. to mean ⊙	Long. ☿	Elongation rel. to mean ⊙	Date of greatest elong.
132 Feb. 2	Ev.	331°	309° 45′	+21° 15′	330° 29′	+19° 46′	Jan. 29
134 June 4	Mo.	48 45	70 0	−21 15	51 9	−19 46	June 10
138 June 4	Ev.	97	70 30	+26 30	97 7	+25 34	June 10
141 Feb. 2	Mo.	283 30	310 0	−26 30	285 44	−25 17	Feb. 2
134 Oct. 3	Mo.	170 12	189 15	−19 3	170 33	−19 39	Sept. 27
135 April 5	Ev.	34 20	11 5	+23 15	34 18	+22 10	April 5
130 July 4	Ev.	126 20	100 5	+26 15	127 14	+26 10	July 6
139 July 8	Mo.	80 5	100 20	−20 15	83 53	−20 20	July 6

to theory (1.15 and 0.95), and the radius of the epicycle to be 0.343. Ptolemy points out that by adding the east and the west greatest elongations in the 1st and 2nd pair, at 70° and 310° longitude, we get 47° 45′, a little bit larger than 2 × 23° 15′ in perigee, confirming his theory that there the distance is least. The 4th pair, with the mean sun at 90° from perigee, was used to demonstrate the geometric details of his model. At that time, with the line from the equant *Q* (Figure 6) to the epicycle's centre *C* perpendicular

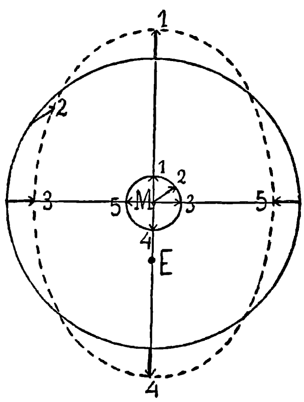

Figure 5

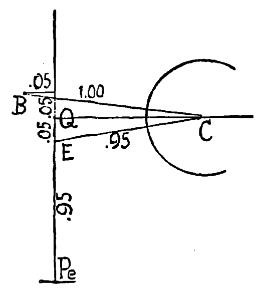

Figure 6

to the line of apsides, the epicycle is seen from the earth *E* under an angle 26° 15′ + 20° 15′ = 2 × 23° 15′, just as large as in perigee; hence the distance *EC* is equal to *EPe, i.e.,* 0.95. Half the difference ½ (26° 15′ — 20° 15′) = 3° determines the distance *QE* = 0.05. Comparing the different lines in the figure we see that indeed the momentary center *B* is at a distance 1.0 from *C*, in agreement with the proposed model.

But this model, however ingeniously constructed, does not agree, as we saw, with reality which does not allow this strongly elongated form. The reason for the discrepancy must be sought in the difficulty and the scarcity of the observations. Mercury can be seen only during short periods in the twilight near the horizon; then it was observed with the astrolabe directed before by means of one of the bright stars, Aldebaran, Regulus, or Antares. A computation of the real longitudes of Mercury from modern data, given in the right hand columns of Table 3, shows, even taking account of all longitudes of Ptolemy being nearly 1° too small, errors up to some degrees; and such errors may spoil all conclusions from equal elongations. Moreover the time of observation often strongly deviates from the true day of greatest elongation, and then the assumption for the case of equal distance to perigee or apogee does not hold. In the 8th elongation Ptolemy has, as Manitius remarked, made an error of 3° in the computation from his own solar table, but it just compensated his error of measurement. So the shortcomings of his theory are easily explained. When, however, we consider that Mercury up to the 17th century, and still later, balked the ingenuity of theorists in representing exactly its motion, we cannot but admire that Ptolemy succeeded in rendering at least the chief features, by means of an inequality of nearly 3° having its maximum at longitudes 100° and 280°. But in the minor details of his artfully designed geometric model his theory was insufficient.

Ptolemy then applies an analogous treatment to a number of old observations of elongations of the years 262-236 B.C., taken from Hipparchus. He derived therefrom that the apogee then had a longitude of 186°, 4° smaller than in his time, so that the apogee took part in the precession of the stars. Considering that a small error in the longitudes produces a twenty times larger error in the apogee, it is evident that this result is entirely fictitious.

V

After the longitudes have been finished the latitudes of the planets must be explained by means of inclinations of the orbits. For the outer planets the excenter, their own orbit, is inclined to the ecliptic, but the epicycle should be parallel to the ecliptic, hence in his model should show the same inclination to the excenter and the same line of nodes. Ptolemy makes a more general assumption: the line of nodes is the same but the angles of inclination are different, to be determined from observation.

For Mars, where the line of nodes is nearly perpendicular to the line of apsides, he observed the latitude at opposition in perigee to be 7°, in apogee 4⅓°. The distances in these points were $0.90 - 0.66 = 0.24$ and $1.10 - 0.66 = 0.44$ times the radius of Mars' orbit. Calling the two inclinations i_1 and i_2, we will have according to Figure 7 $i_1 + (0.66/0.24)\, i_2 = 7°$ and $i_1 + (0.06/0.44)\, i_2 = 4\tfrac{1}{3}°$; hence $i_1 = 1°$, $i_2 = 2° 15'$. Are they really different? If the observed latitudes had been slightly different, 6⅔° and 4½° (the true values are 6° 43′ and 4° 37′), both inclinations would have been found nearly equal, about 1°.8 (the true value is 1° 51′).

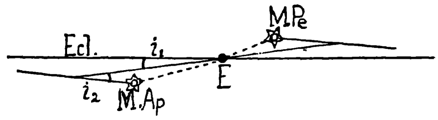

FIGURE 7

For Jupiter and Saturn "we did not find a difference in latitude in the oppositions in perigee and apogee." (Book XIII, Chapter 3). Hence he compares the latitude in conjunction and in opposition. The data he makes use of look very coarse: for Jupiter 1° and about 2°, for Saturn 2° and about 3°. From the distance ratios then he finds: for Jupiter $i_1 = 1\frac{1}{2}°$, $i_2 = 2\frac{1}{2}°$ (true $i = 1° 19'$), and for Saturn $2\frac{1}{2}°$ and $4\frac{1}{2}°$ (true $i = 2° 30'$). The greatest northern deviation he takes to occur at longitudes +20° and —50° different from apogee, hence the ascending nodes for Mars, Jupiter, Saturn are 25°, 91°, 93° (true values 35°, 81°, 98°).

For Venus and Mercury matters should be easy because here only the epicycle has to be inclined to the deferent which coincides with the ecliptic. But the complicated structure of his model is made still more difficult by the introduction of his inclination-mechanism. As a main feature he has, of course, an inclined epicycle, for Venus $2\frac{1}{2}°$ with the ascending node at 55° coinciding with apogee (real values 3° 24′ and 59°), for Mercury $6\frac{1}{4}°$ with the node at 10° coinciding with perigee (real values 7° 0′ and 26°). But he adds an oscillation of the deferent, for Venus between 0° and $+\frac{1}{6}°$, so that the centre of the epicycle remains always at the northern side, for Mercury between 0° and $-\frac{3}{4}°$, so that here it is always at the southern side. This superfluous complication, of course, is too small to be secured by the rather rough observations of latitude.

Ptolemy points out that such a model as he devised for the planetary orbits requires an additional mechanism. In the epicycle theory the epicycle is assumed to be fixed, as it were, on the deferent, so that in revolving along the deferent the outer point remains at the outside; if in an inclined epicycle this point is highest above the plane of the deferent it remains highest while revolving, so that the line of nodes continually turns around. In order to keep the epicyle always in the same situation in space, Ptolemy introduces a special contrivance; the innerpoint of the epicycle is situated upon a small vertical circle having its centre in the ecliptic, so that in the period of revolution this inner point is oscillating up and down between the extreme northern and southern deviation. It may look somewhat artificial and mechanical, but it works. To make it still more perfect it is made as eccentric as the deferent itself, so that even the small variations of velocity in longitude and latitude keep pace. And then Ptolemy concludes with the following philosophical reflection.

"And let nobody, looking at the imperfection of our human contrivances, regard the hypotheses here proposed as too artificial. We may not compare human things with things divine nor, in order to demonstrate such enormous things, make use of entirely incomparable examples. For what indeed is more dissimilar than beings always behaving in the same way and creatures never doing so? What more dissimilar than creatures who may be disturbed by any trifle and beings that will never be disturbed, not even by themselves? Of course we must try as far as possible to adapt the simpler hypotheses to the heavenly motions; if, however, we cannot succeed in this way, we must go to hypotheses allowing of such an adaptation. For if once all celestial phenomena can be explained on the basis of the hypotheses, why then should anybody wonder at the possibility that such a complicated interlocking should be inherent to the motions of the heavenly bodies, since they are not subject to any constraint of nature. . . The simplicity itself of the celestial processes may not be judged according to what is held simple among man, the more so since on earth there is no unanimity about what is simple. For if we should look at it from this human point of view, nothing of all that happens in the celestial realms would appear to us to be simple at all, not even the very immutability of the first (daily) rotation of heaven, because for us human beings this very unchangeableness, eternal as it is, is not only difficult, but entirely impossible. But in our judgment we have to proceed from the immutability of the beings revolving in heaven itself and of their motions; for from this point of view they would all appear to be simple, and even simple in a higher degree than what on earth is regarded as such, because no trouble and no pains can be imagined with regard to their wanderings." (Book XIII, Chapter 2).

VI

With the numerical derivation of the elements of the planetary orbits the task was not finished. They had to be adapted to practical use, by means of tables. In the 9th book tables are given, and their practical use is explained, for the mean motions of the planets in longitude and in anomaly, *i.e.*, along the epicycle; in the 12th book the first and the second inequality (for eccentricity, and for epicycle) are given; and the 13th book contains tables for the latitudes. Moreover tables and directions for computation are given for remarkable phenomena, for the turning points between direct and retrograde motion, as well as, in the case of Mercury and Venus, for the greatest elongations. Finally tables are added to derive the heliacal risings and settings of the planets, which played an important role in the first observations of ancient astronomy. Thus the foundation was laid upon which in four following books—afterwards usually separately published under the name "Tetrabiblon"—the theory of astrology, the doctrine of the relation between heavenly and earthly phenomena could be erected.

VII

In this summary of Ptolemy's exposure of the planetary motions we ever and again met with modes of treatment that looked uncommon or even suspicious, at variance with our ideas on scientific work. Data exactly equal that never could have been given thus by his rather coarse observations, as well as good results derived by good geometrical computation from fictitious differences in observed quantities, have sometimes raised the suspicion, or even the accusation that he had simply invented his observations, or at least had forged or doctored them in order to make them harmonize with his theory. So we read in Delambre's great work "Histoire de l'astronomie ancienne, 1817," in the "Discours Préliminaire."

"Has Ptolemy himself observed? The observations which he tells us he has made, are they not computations from his Tables, and examples for the use of better understanding of his theories?" (p. xxxv). "As to the chief question, we see no means to decide it. It seems hard to deny absolutely that Ptolemy has made observations himself, at least if he did not find elsewhere what he wanted for his researches. If he had in his hands observations in greater number, as he says himself, we may reproach him that he did not communicate them and that nowhere he tells what might be the possible errors of his solar, lunar, and planetary tables. An astronomer who today should act in this way would be certain to inspire no confidence at all. But he was alone, he had no judges and no rivals; a long time he has been admired on his own word; and at present it is not deemed worth while to compute the few observations he has left. . ." (p. xxxiv-xxxv).

Delambre here explicitly proclaims the methods of his time to be the standards for judgment of scientific work for all centuries. It is clear, however, that we cannot judge Ptolemy's work according to the ideas and habits of modern scientific research, and that we come to a false and unjust appreciation if we do. We have to place ourselves entirely in the ways of thinking of antiquity, and to realize their differences from our scientific practice. This holds in the first place for the relation of experience, observation to theory.

There is no better way to perceive this difference than another work of antiquity, the small writing of Aristarchus "On the Sizes and the Distances of Sun and Moon," the only one of his works that still exists. From the observation that the moon is exactly divided in two equal halves when its angular distance from the sun is 3° less than a right angle, he deduces that the sun is 19 times more distant than the moon. We should wish to know by what observations he arrived at this amount of 3°, on which the result depends; any pupil of a secondary school then immediately infers the ratio 19. In Aristarchus' book we find at the start six "hypotheses," each some few lines only, of which the third reads: "When the moon appears to us halved its distance from the sun is less than a quadrant by one-thirtieth of a quadrant." That is all. Thereupon 14 pages follow with geometrical demonstrations to derive that in this case the sun is less than 20 times and more than 18 times as far remote as the moon.

The theoretical apparatus in scientific research, the mathematical methods nowadays at our disposal, have acquired so high a degree of perfection, we have imbibed them at school already so thoroughly that the working up of the data of experience offers no difficulty at all. Hence all attention of modern astronomers is directed to observation, to its errors and uncertainties. Their communication of results bears the character of a protocol, of precise authentic statement of facts, with circumstances and details, which, once given cannot be changed any more. Research of nature is a socially organized and standardized world-wide business with elaborated patterns and acknowledged importance. In the ancient world, however, it was the first cautious stepping into an unknown field, a sublime personal work of pleasure. Empiricism was chiefly the all-day practice (*i.e.*, of sun and moon), which in such personal occupations broadened into more attentively looking at uncommon things (*e.g.*, the planets). Theory was the new world of wonder beyond the common knowledge, it was philosophy searching wisdom, eager to discover the essence of things and to unveil world-structure.

So observations in ancient science had no protocol-character; the idea of errors of observation as a normal character, determining their treatment and asking for elimination, played no role. When it was necessary one looked or measured where the planet hung out. The figures indicating its position were simple facts. Ever again we read in Ptolemy, after his mentioning the observations: these numbers we accept as given. And then follows the geometrical demonstration, taking a broad space, how to derive from these data by means of exact propositions and computations the desired elements of the orbit. There does not appear any concern about the reliability and accuracy of the figures used or about its influence upon the values derived. Geometric demonstration is paramount, and this supremacy goes so far as to derive correct results by applying a well-devised method to fictitious data. Again Aristarchus' little work may give a hint as to the underlying thought by calling the empirical data at the outset "hypotheses," *i.e.*, assumptions from which the real, the geometrical work has to start.

The outcome of the work is a geometrical picture of the world of celestial bodies. It is a picture of eternal continuous motion in circular orbits, obeying determinate laws, a picture full of simple harmony, a "cosmos," an ornament.* It is expounded in a straight progression of exact demonstrations, without disturbing irregularities. When the line of apsides of Venus is derived from equal elongations at both sides,

*The Greek word "kosmos" means ornament.

these for the strict demonstration had to be exactly equal. Had they been a little bit different, an explanation to the reader would have been needed that it made no essential difference; this digression would have distracted the attention, left a certain feeling of doubt, and disturbed the harmony of the construction. The book, moreover, the work of the author, had to conform to a standard of perfection in form and workmanship. Spiritual work in those times was not so different from handicraft; just as the craftsman removed carefully from his product any rough irregularity that could disturb the eye in enjoying the pure harmony of form and line, so the mental piece of work had to captivate eye and mind in the pure presentation of the universe in its mathematical image.

Geometry in Greek culture occupied a paramount place as the only abstract and exact science. For its rigidly logical structure, proceeding from axiom and proposition to proposition it stood out as a miracle of the human mind, a monument of abstract truths, entirely outside the material world, and notwithstanding its visibility to the eye entirely spiritual. The small bit of utility in its origin from Egyptian geodesy hardly carried weight against Euclid's great theoretical structure. Surely the "sphaerica" too, the theory of the sphere and its circles found a wide practical application in astronomy in the description of the phenomena of the celestial sphere, the risings and settings of the stars, and so was used and taught during all following centuries. But spherics was a small part only of geometry. The entire science of lines and angles, of triangles, circles, and other figures with their relations and properties was a purely theoretical doctrine, studied and cultivated for its inner beauty.

Now, however, came the science of the planetary motions, the work of Ptolemy, as a practical embodiment of the theory. What else would have been imagined truths, existing in phantasy only, here in the structure of the universe became reality. Here it acquired form and specificness, definite value and size. In the world of planets the circles moved, distances stretched and shrank, angles widened and dwindled, and triangles changed their form, in endless stately progress. If we call Greek astronomy the oldest, indeed the only real science of antiquity, we must add that it was geometry materialized; the only field, truly, where geometry could materialize. Whereas without this world the practice for students of geometry would have been restricted to an idle working with imaginary self-constructed figures, they found here a realm, the sole but the grandest, where their figures were real things, where they had definite form and dimension, where they lived their own life, where they had meaning and content, the orbits of the celestial luminaries. Thus the "Mathematical Composition" was a pageant of geometry, a celebration of the most profound creation of human mind in a representation of the universe. Can we wonder that Ptolemy in a