Eliminating M, R and T'_c we obtain:

$$\rho_c \sim \left(\frac{\beta_c \mu_c}{1-\beta_c}\right)^{\frac{s}{s+3}} \cdot \varepsilon_c^{\frac{3}{s+3}} \quad \ldots\ldots\ldots (22)$$

If now μ_c and ε_c have the same value in both stars (22) reduces to:

$$\rho_c \sim \left(\frac{\beta_c}{1-\beta_c}\right)^{\frac{s}{s+3}} \quad \ldots\ldots\ldots\ldots (23)$$

So the stars of larger mass in the critical stage have the smaller central densities. Now at smaller densities KRAMERS' law is more nearly satisfied than at larger densities. Consequently the exponent p in (10) will be larger in stars of large mass. So it seems to be true again, that large stellar mass favours the formation of extended envelopes.

6. *Conclusion.*

In the above it is shown, that there are some circumstances which may favour the formation of extended envelopes around the central part of a star. All these circumstances may be reduced to two, viz. large stellar mass and the existence of a core deprived of hydrogen.

If the giant stars really represent such composite configurations they are in the final stage of their evolution, a view, which is exactly contrary to the opinion generally held. More detailed discussions will be necessary, however, to settle the question. In these discussions an accurate calculation of the absorption coefficient will be of great significance.

A remarkable place in COPERNICUS' „De Revolutionibus", by *A. Pannekoek* [1]).

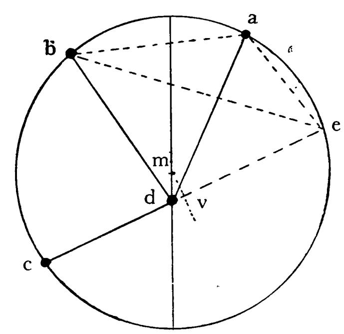
There is a curious place in COPERNICUS' *De Revolutionibus* that seems to have escaped attention. It relates to the computation of the orbit of Jupiter.

COPERNICUS in his planetary theory assumes circles with constant velocity. In order to represent the variable apparent velocity he supposes a small epicycle on which the planet is moving, the radius of which is $^1/_3$ of the eccentricity of the sun in the planet's orbit; this affords, in first order terms, the same variations as PTOLEMY's theory of the centre of the circle being situated midway between the earth and the punctum aequans.

The problem of how to derive the orbit (i.e. eccentricity and longitude of apogaeum) from three longitudes at three epochs (of opposition) with known intervals, meets with the same difficulty that PTOLEMY had to face. It cannot be solved directly but only by successive approximations in some few steps. First a simple eccentricity in a constant circular motion is assumed; then corrections to the observed longitudes are computed for the epicycle, and the computation is repeated. The simplified problem, here as with PTOLEMY, is identical with the geodetic problem solved by SNELLIUS and by POTHENOT: of finding the situation of an unknown point between three known points, when the directions to them have been measured. The three known points are the places of the planet in its circle, which are known since the arcs between them are proportional to the elapsed time; the directions are known as the observed longitudes of opposition. As an example of the process of computation, the same as used by PTOLEMY, we take Saturn's orbit from three oppositions observed by COPERNICUS himself.

1) Received in October 1945.

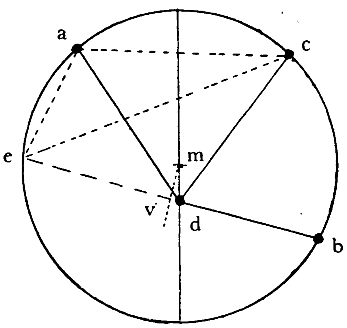
FIGURE 1.



In the figure a, b, c are the planet's places; d is the place of the sun, m is the centre; the data are arc $ab = 75°39'$; arc $bc = 88°29'$; $\angle\, adc = 154°43'$; $\angle\, bdc = 86°42'$. In triangle ade the angles are known ($d = 25°17'$, $a = 72°39'$, $e = 82°4'$), hence the ratio of the sides may be computed ($ae/de = 0{\cdot}4474$); the same holds for triangle bde ($d = 93°18'$, $b = 42°72'{\cdot}5$, $e = 44°14'{\cdot}5$, $be/de = 1{\cdot}4787$). Then in triangle abe two sides (in unit de) are known as well as the inclosed angle $e = 37°50'$; so the other angles may be found ($a = 128°28'$, $b = 13°42'$); they afford the sides expressed in the radius as unity ($be = 1{\cdot}5657$, $ae = 0{\cdot}4737$), so in the same unity we have $de = 1{\cdot}0587$. Since moreover arc ae ($2 \times \angle\, abc$) added to arc abc gives $191°32'$, i.e. $2 \times 5°46'$ more than the semicircle, we have $mv = 0{\cdot}1005$ and $ev = 0{\cdot}9950$, hence the difference $vd = 0{\cdot}0637$. These distances determine the eccentricity $md = 0{\cdot}1191$ and $\angle\, dmv = 32°25'$, from which the longitude of the aphelium is found. This is the first approximation, from which the corrections are derived; then the computation is repeated for the next approximations.

For Jupiter COPERNICUS uses the following data:

FIGURE 2.



arc ac = 94°10′, arc cb = 66°10′, ∠ cdb = 65°10′, ∠ adb = 151°54′. In triangle cde the angles are known (d = 114°50′, e = 33°5′, c = 32°5′), as well as in triangle ade (d = 28°6′, e = 80°10′, a = 71°44′); so he finds ce/de = 1·8150/1·0918 and ae/de = 0·9420/1·8992, and working along in the same way his results are md = 0·1193 and ∠ dmv = 36°35′. Then he proceeds: "The which certainly badly fits the phenomena, since the planet does not move along the proposed eccenter, so that this method of demonstration resting on an uncertain principle, cannot lead to anything certain ... And in no other way was it possible to construct the equable and the apparent motion of Jupiter through these three given points ... than by following the total deviation of eccentricity of the centres given by PTOLEMY ..." [1])

This is a most astonishing utterance, since there is nothing to be objected in the method, which in all other computations has been used with good results. How was it that COPERNICUS here gave it up as unreliable? Doubtless because he saw the great difference between his eccentricity 0·1193 and PTOLEMY's 0·0917 he needs suspected that there must be something wrong.

If we carry through the computation we find from triangle cde (∠ d = 114°50′, ∠ e = 33°5′, ∠ c = 32°5′) the ratio ce/de = 1·7090, and from triangle ade (∠ d = 28°6′, ∠ e = 80°10′, ∠ a = 71°44′) the ratio ae/de = 0·4960. Then in triangle aec with ∠ e = 47°5′ we find ∠ a = 118°4′, ∠ c = 14°50′, hence expressed in the radius as unity we have ce = 1·7650, ae = 0·5123, de = 1·0327. From arc ae = 29°40′, added to arc bca = 160°20′ we find arc $bcae$ = 190°0′, by 2× 5°0′ surpassing the semicircle; then mv = 0·0872, bv = 0·9961, hence dv = 0·0366, and the eccentricity md = 0·0945 and ∠ vmd = 22°46′. This agrees sufficiently with PTOLEMY's value which he adopted. It shows that there must be an error in his computations. This error consisted in his interchanging the angles c and e in the first triangle, so that for c he took the value 33°5′ in stead of 32°5′ and he found ce/de = 1·8150/1·0918 = 1·662 in stead of 1·8150/1·0622 = 1·709. This kind of error was easily overlooked in a control afterwards; it corresponded to having observed the longitude of the opposition 2° wrong, and so led to a value of the eccentricity which he rightly dismissed as impossible.

That such an error was made and not detected afterwards can easily be understood. It is more curious, indeed, that during four centuries, in which hundreds of students of astronomy certainly carefully studied COPERNICUS' work, none of them took the trouble of repeating his computations, especially where the very words of the author indicated that something was wrong.

[1]) "Quae nimirum parum conveniunt apparentiis non currente planeta per propositum eccentrum, ut neque modus hic demonstrationis in incerto nixus principio certum quid possit adferre ... Nec aliter Jovis motum aequalitatis et apparentiae possibile erat componere in his tribus terminis propositis ... nisi sequeremur totam centrorum egressionem excentricitatis a PTOLEMAEO proditam ..."

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