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Intensity and Selfabsorption of Chromospheric Lines, by *A. Pannekoek*.

1. In the discussion of the results of our observations of the flash spectrum at the Lapland eclipse the problem was posed to compute the real intensity of chromospheric lines. Especially it was the question in what manner the absolute and the relative intensity of different lines is changed by selfabsorption.

In the following computation we suppose the density of the chromospheric gases so small that the effect of collisions may be neglected, so that absorption and emission of radiation is the only method of energy transfer. The atoms are exposed to the photospheric radiation from the hemisphere below; we assume that they absorb only a small fraction of it so that up to the highest layers this radiation does not sensibly diminish. This implies the supposition that the Fraunhofer absorption has already taken place in the lower reversing layer.

In the case of two quantum states (number of atoms n_1 and n_2) we put the transition probabilities $B_{12}J$ and A_{21} for incident radiation J ; the stimulated transitions 2-1 may be neglected if in the Planck function the second term in the denominator is omitted. Then from

$$J = \beta \sigma e^{-h\nu/kT_1} \quad \text{and} \quad A_{21} = \sigma B_{12}$$

($\sigma = 8\pi h\nu^3/c^2$ and $\beta = 1/2$ in this case)

follows
$$n_2 = \beta n_1 e^{-h\nu/kT_1}, \tag{1}$$

where T_1 is the black radiation temperature of the photospheric emission, and ν denotes the frequency corresponding to the transition 1-2. The radiation which is emitted in a certain horizontal direction by the atoms of a volume element unity is then given by

$$E = n_2 A_{21} h\nu d\omega.$$

The horizontal stream of energy is diminished on the path dx by the amount $n_1 B_{12} E h\nu dx$ and increased by the addition of $E dx$; thus for the case of n_1

constant along the path the resulting horizontal intensity will be

$$\int n_2 A_{21} h\nu e^{-n_1 B_{12} h\nu x} dx d\omega.$$

Here $B_{12} h\nu = \alpha$, the atomic absorption coefficient; putting $\alpha n_1 = \kappa \rho$ (where $\rho = nm$ is the density, $\kappa = \alpha/m$, n_1/n is the mass absorption coefficient, m denotes the mass of an atom, n the total number of atoms in 1 cc.) and $P = \sigma e^{-h\nu/kT_1}$ (the Planck function) we find, making use of the relation (1), for the outward going intensity

$$I = \int \kappa \rho \beta P e^{-\kappa \rho x} dx d\omega \tag{2}$$

2. In the case of more than two quantum states this formula needs a small modification. Then the ratios n_2/n_1 and n_3/n_1 cannot be computed from equilibrium conditions; the number of transitions between two states to and from are not equal, and $\gamma_{12} = n_2 A_{21}/n_1 B_{12} J$ is different from 1. For a series of cyclical transitions 1-2-3-1. S. ROSSELAND has shown (*Ap. J.* 63, 231) that

$$\gamma_{13}/\gamma_{12}\gamma_{23} = \beta F_{13}/F_{12}F_{23},$$

where the factors F are nearly = 1 and $\beta = 1/2$. From this result, which may be interpreted in this way that in a field of non isotropic radiation for the large transitions the absorption is increased, for the intermediate transitions the emission is increased, we may infer that $\gamma_{13} < 1$, $\gamma_{23} > 1$, $\gamma_{12} > 1$. From the above formulae we find now

$$n_2/n_1 = \gamma_{12} e^{-h\nu_1/kT_1}$$

and
$$I = \int \kappa \rho \gamma P e^{-\kappa \rho x} dx d\omega. \tag{2^*}$$

Because we have assumed the density ρ constant along the beam of light we find, if the atmosphere of this gas extends from $x = -1/2 l$ to $+1/2 l$,

$$I = \gamma P (1 - e^{-\kappa \rho l}) d\omega. \tag{3}$$

3. The spectral line has a finite width, i. e. for a certain extent of closely adjacent λ the value of κ is different from zero, decreasing from its maximum value κ_0 at the centre to zero at both sides. We assume $\kappa = \kappa_0 \varphi(u)$ to be a known function of $u = (\lambda - \lambda_0)/b$, where the parameter b (the mean width of the line) is so chosen that $\int \varphi(u) du = 1$, thus $\int \kappa d\lambda = \kappa_0 b$. This value must now be substituted for κ in the formula $\kappa = B_{12} h \nu n_1 / n m$.

The total intensity of the outward going radiation is now

$$I = \gamma P \int (1 - e^{-\kappa_0 \rho l \varphi(u)}) d\lambda = \gamma P b f(y), \quad (4)$$

if we put

$$\kappa_0 \rho l = y; \quad \int (1 - e^{-y \varphi(u)}) du = f(y) \text{ (limits } \pm \infty) \quad (5)$$

If the function $\varphi(u)$ giving the distribution of light in the spectral line is known, the function $f(y)$ may be computed. The meaning of this formula is that, while for a very thin sheet the profile of the emission line is given by $\varphi(u)$, for thick layers its top is flattened, and the light of this flattened curve is integrated.

4. To find the total light of the chromosphere (over a small breadth, e. g. $1'$ of arc) the light of each sheet must be integrated over the full height of the atmosphere. We first take a homogeneous mass of gas, limited by a sphere, with $\rho = \text{constant}$; calling the vertical coordinate z , we have $l^2 = 8 R z$. Then

$$I = \int_0^{z_0} \gamma P b f(y) dz = \frac{\gamma P b}{4 \kappa_0^2 \rho^2 R} \int_0^{y_0} y f(y) dy.$$

Thus putting

$$\frac{3}{y_0^2} \int_0^{y_0} y f(y) dy = F_1(y_0); \quad y_0 = \kappa_0 \rho \sqrt{8 R z_0} \quad (6)$$

we have

$$I = \gamma P b \cdot \frac{2}{3} z_0 F_1(y_0). \quad (7)$$

Since the total mass of the atmosphere above the tangent line at $z = z_0$ is $M = \frac{2}{3} z_0 y_0 / \kappa_0$ we may also write

$$I = \gamma P b \kappa_0 M \frac{F_1(y_0)}{y_0}. \quad (7^*)$$

For a very thin sheet of gas $1 - e^{-\kappa \rho l}$ reduces to $\kappa \rho l$; then $f(y) = \int y \varphi(u) du = y$ and for one layer $I = \gamma P b y$; in this case $F_1(y_0) = y_0$ and for the whole atmosphere $I = \gamma P b \kappa_0 M$. The factor $F_1(y_0)/y_0$ thus represents the rate of selfabsorption of the line, just as the factor $f(y)/y$ does for a single atmospheric layer.

5. Next we consider the case of an atmosphere with exponential decrease of density, such as corresponds to a partial support by radiation pressure.

Along each horizontal line of sight we compress the atmospheric layers to the density of the central tangent point A ; then $d\xi = dx \cdot \rho / \rho_0$. Now $\rho = \rho_0 e^{-z/h}$, where h , the aequivalent height of the atmosphere $= HT/g$.

Since $z = x^2/2R$ we have $d\xi = dx e^{-x^2/2Rh}$; thus the limits $\pm \xi$ on a line of sight along which the constant density has the value ρ_0 of the point A will be $\pm \frac{1}{2} \sqrt{2\pi R h}$, and the thickness l (art. 3) of the mass of constant density is in this case

$l = \sqrt{2\pi R h}$; it may be assumed constant for all layers. Thus the results of art. 2 and 3 may be used also in this case.

Integrating now for z we have to take $\int \gamma P b f(y) dz$ where in $y = \kappa_0 \rho l$ the factor ρ varies with z . Because $d\rho/\rho = -dz/h$ and $d\rho/\rho = dy/y$, we find

$$I = \gamma P b \int_0^{y_0} \frac{h f(y)}{y} dy$$

Putting

$$\int_0^{y_0} \frac{f(y)}{y} dy = F_2(y_0),$$

where

$$y_0 = \kappa_0 \rho_0 l = \kappa_0 \rho_0 \sqrt{2\pi R h},$$

we get

$$I = \gamma P b h F_2(y_0). \quad (9)$$

The total mass of this atmospheric segment, situated above the tangent line of sight, is given by

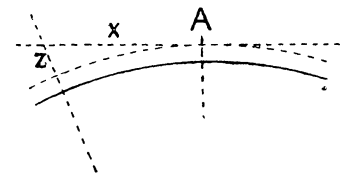
$$M = \int_0^{\rho_0} l \rho dz = \int_0^{\rho_0} h l d\rho = h l \rho_0 = h y_0 / \kappa_0;$$

thus

$$I = \gamma P b \kappa_0 M \frac{F_2(y_0)}{y_0} \quad (9^*)$$

Again the last factor, depending on y_0 , represents the decrease of the total emission of these atoms due to selfabsorption. This emission will be $\gamma P b \kappa_0 h y_0 = \gamma P b \kappa_0 \rho_0 h l$, where $y_0 = \kappa_0 \rho_0 l$ may be called the optical depth at the basis of the visible chromosphere, and h the height of an aequivalent atmosphere of constant density ρ_0 . We compare two elements with very different h ; then they may have the same real emission, if y_0 is larger for the gas which is more condensed in the lower layers; but the selfabsorption is larger in the latter case and the observed intensity is smaller.

FIGURE 1.



6. The functions $f(y)$, $F_1(y_0)$ and $F_2(y_0)$ depend on $\varphi(u)$ giving the distribution of intensity over the width of an emission line. For a gas of extreme tenuity, where collisions practically play no part, while the temperature is rather high, the chief agency is the Doppler broadening by the molecular velocities. In this case we have

$$\varphi(u) = e^{-\pi u^2} \quad (10)$$

$$f(y) = \int (1 - e^{-ye^{-\pi u^2}}) du = \frac{1}{\sqrt{\pi}} \int (1 - e^{-ye^{-v^2}}) dv \quad (11)$$

With the aid of a table, giving $1 - e^{-a}$ as a function of a the functions f , F_1 and F_2 have been computed by mechanical integration. In the following tables the values of F_1 and F_2 are given, as well as the factors of selfabsorption of form. (7) and (9) for these cases.

In the case of a chromosphere wholly supported by radiation pressure, where the decrease of density is much slower than an exponential decrease, the results may be assumed to be between the values F_1 and F_2 .

y_0	$F_1(y_0)$	$F_1(y_0)/y_0$	$F_2(y_0)$	$F_2(y_0)/y_0$	y_0	$F_1(y_0)$	$F_1(y_0)/y_0$	$F_2(y_0)$	$F_2(y_0)/y_0$
0	0	1	0	1	8	2.346	0.293	3.531	0.441
0.1	0.097	0.975	0.098	0.983	9	2.424	269	3.744	416
0.2	190	949	193	966	10	2.491	249	3.939	394
0.3	278	925	285	950	12	2.602	217	4.285	357
0.4	361	903	374	934	14	2.692	192	4.588	328
0.5	440	881	460	919	16	2.768	173	4.856	303
0.6	516	860	543	904	18	2.832	157	5.097	283
0.7	588	839	623	890	20	2.888	144	5.317	266
0.8	656	820	701	877	25	3.003	120	5.793	232
0.9	721	801	777	864	30	3.093	103	6.194	206
1.0	784	784	851	851	35	3.167	0905	6.540	187
1.2	900	750	992	826	40	3.229	0807	6.846	171
1.4	1.006	719	1.125	804	50	3.330	0666	7.367	147
1.6	1.103	690	1.251	782	60	3.410	0568	7.804	130
1.8	1.192	662	1.371	762	70	3.476	0500	8.179	117
2.0	1.275	637	1.485	743	80	3.532	0441	8.510	106
2.5	1.454	582	1.748	699	90	3.580	0398	8.805	0978
3.0	1.603	534	1.983	661	100	3.623	0362	9.073	0907
3.5	1.729	494	2.196	628	120	3.696	0308	9.542	0795
4.0	1.836	459	2.391	598	140	3.756	0268	9.945	0710
4.5	1.929	429	2.569	571	160	3.808	0238	10.299	0644
5.0	2.010	402	2.735	547	180	3.852	0214	10.614	0590
6	2.146	358	3.033	505	200	3.891	0195	10.900	0545
7	2.255	322	3.295	471					

7. The absorption coefficients $\kappa_0 b = B_{12} h \nu n_1 / n m$ contain two factors, the fraction of the atoms in state 1 n_1/n , and the number of transitions B_{12} per second for each of these atoms. For transitions belonging to the same multiplet these probabilities B have constant ratios not depending on physical conditions. The researches at the Utrecht Physical Laboratory made by BURGER, ORNSTEIN, DORGELO and others have shown that in normal cases these ratios may be expressed by simple numbers; on the basis of their sum-rule theoretical expressions for the relative intensities of the components of a multiplet have been derived by RUSSELL, HÖNL, KRONIG and others. For the lines constituting a multiplet our y_0 are proportional to these known values; the observed I in the chromosphere are proportional to $F(y_0)$; thus by plotting them we

may find the real values of y_0 , the factors of self-absorption $F(y_0)/y_0$ and the true total emission $\gamma P b h y_0$ of each line.

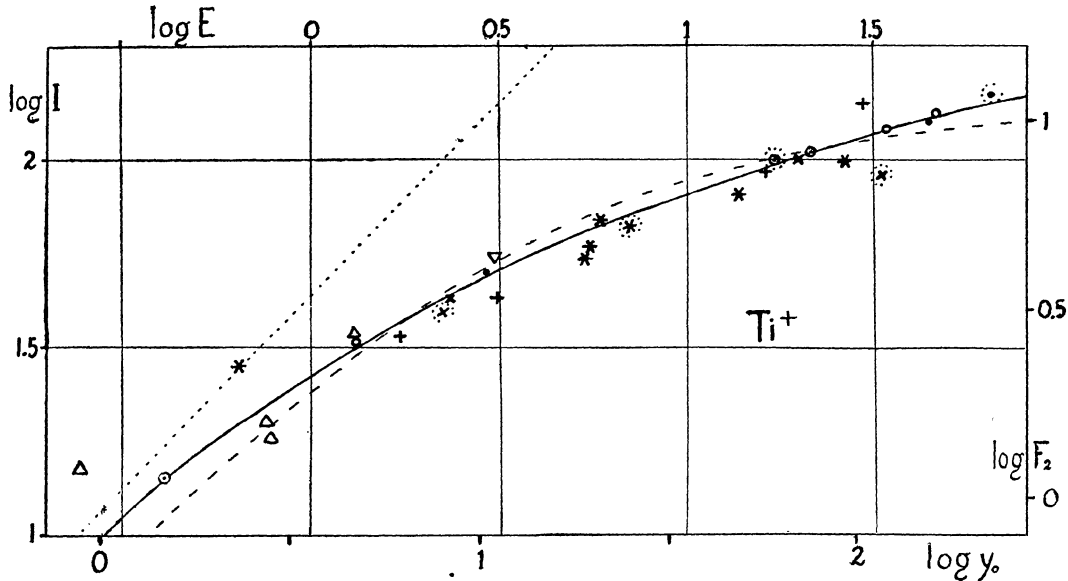
In our discussion of the flash spectrum (A. PANNEKOEK and M. G. J. MINNAERT, *Results of Observations of the Total Solar Eclipse of June 29 1927, I: Photometry of the Flash Spectrum* *) we have given two diagrams (for $Fe +$ and $Ti +$, *Results* p. 93 and 94) of $\log I$ and $\log E$ (theoretical intensities of the multiplet components multiplied by empirical coefficients in order to bring them upon one curve). Through the plotted data we have drawn empirical curves, which now must be brought into coincidence with the

*) Verhandelingen der Kon. Ak. v. Wet. Amsterdam, Vol. XIII, 1928, called "*Results*" in this note.

theoretical curves $F(y_0)$. It appears that the empirical curves are less curved; especially the curvature of $F_1(y_0)$ is rather too large to represent the data. The curve $F_2(y_0)$, however, does not leave much larger deviations than the empirical curves; it is represented

by a full line in the figures on this page (on one of them F_1 is represented by a line of dashes). From the position of the data on this diagram we find that e. g. for the lines 4450 and 4444 of $Ti+$ ($I = 50$ and 128×10^{20} , relative intensity 1:14, $\log E = 0.49$ and

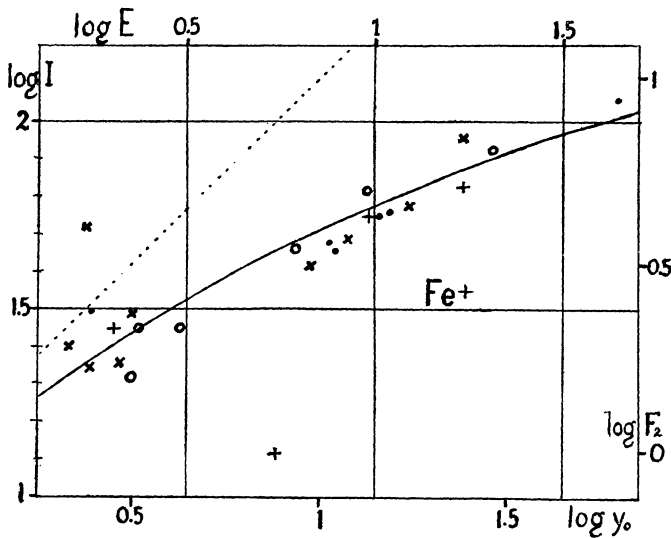
FIGURE 2.



1.64), $\log y_0 = 1.04$ and 2.19 , $y_0 = 11$ and 155 ; for these arguments our table gives the reduction coefficient of selfabsorption 0.37 and 0.066; dividing the observed intensities I by these coefficients we get

for $Ti+$ $\log y_0 = \log E + 0.55$
 $\log F_2(y_0) = (\log I - 20) - 1.10$
 for $Fe+$ $\log y_0 = \log E + 0.15$
 $\log F_2(y_0) = (\log I - 20) - 1.12$

FIGURE 3.



the real emissions $I_r = 135$ and 1940×10^{20} . In a more general way these results may be deduced from the relations between the two kinds of scales (at the bottom and the top, and on the left hand and the right hand side of the diagram):

Since $I_r = I_0 \times y_0 / F_2(y_0)$ we find
 $\log I_r - 20 = \log E + 1.65$ $\log I_r - 20 = \log E + 1.27$

Thus the real emission of each wavelength is found by multiplying its E by 45×10^{20} (for $Ti+$) and 19×10^{20} (for $Fe+$); the same holds for the total intensities of the multiplets (in *Results*, Table X p. 95 for $Ti+$ and on p. 93 for $Fe+$).

8. From the values of the emission found in this way it is possible, with the aid of some approximate assumptions, to deduce the intensities of the absorption lines, which these atoms will produce in the solar spectrum. The values deduced represent the emission of a section of the chromosphere, whose breadth B appears as $1'$ of arc. (in $\text{cm } \log B = 9.64$), over a solid angle $d\omega = 1$; thus the total emission is found by multiplying by 4π . It is emitted by a mass of atoms $\rho_0 B h l$; the mass of the atoms contained in a vertical column with basis 1 cm^2 is $\rho_0 h$; thus its emission is found by dividing $4\pi \times I_r$ by $B l$. The equivalent depth $l = \sqrt{2\pi R h}$ is not exactly known, because we do not know h , the equivalent height of the $Ti+$ atmosphere; from the fact, that $Ti+$ has

been found up to a height of 4000 km, we may assume h to be between 100 and 1000 km (differing a factor 3 in l), and we will make our computations with $h = 500$ km; then $\log l = 9.67$. For the strong Ti + line 4444, from $\log I_2 = 23.29$, $\log 4\pi I_r = 24.39$, and $\log Bl = 19.31$ we get for the emission of a vertical column of atoms of 1 cm^2 basis 5.08 ergs. The same amount of energy has been absorbed by these atoms from the solar radiation, which we may assume to be concentrated in a vertical stream. The solar radiation of 1 cm^2 , contained in $d\lambda = 1 \text{ AU}$ at $\lambda 4500$, amounts to $2\pi \times 4.3 \times 10^6$ ergs (*Results* p. 31), $\log = 7.43$; thus the fraction of this energy, absorbed by the chromospheric atoms, is 0.004 ($\log = 5.08 - 7.43 = -2.35$); or, expressed in another way, these atoms will produce an absorption line of aequivalent width 0.004 AU. This value should be somewhat increased, because in our flash negative the lowest parts of the chromosphere were covered by the moon during part of the exposure. Now the intensity of the Fraunhofer line 4443.8, being 5 in ROWLAND'S scale, corresponds to an aequivalent width of 0.24 AU. Thus it is shown, that the chromospheric gases produce only a small part of the total absorption in the solar atmosphere. This is in accordance with the views of MILNE, who finds that the main part of the absorption of a Fraunhofer line takes place in photospheric layers below the chromosphere.

9. Two different numerical results have thus been obtained from these data for each spectral line: the absorption coefficient for a certain central wavelength $\kappa_0\rho_0$, and the amount of absorption over the whole width of the line, deduced from the amount of emitted energy and the incident solar radiation. This solar radiation of the wavelength λ_0 in the centre of the line has already been weakened by absorption in the

lower photospheric layers; we may assume for it now the observed intensity in the centre of the corresponding Fraunhofer line. From the data given *Results* p. 44 (width after VON KLÜBER, aequivalent width after MINNAERT) we find for a line of intensity 5 an absorption of 60%, i. e. a reduction to 40% of the background. From $\log \kappa_0\rho_0 l = \log \gamma_0 = 2.19$

we find (by $\log l/h = 1.98$) $\log \kappa_0\rho_0 h = 0.21$; from $\log \gamma P b \kappa_0\rho_0 h = 5.08$ and $\log 0.40 P = 7.03$ we find $\log \gamma b \kappa_0\rho_0 h = -1.95$; combined they give $\log \gamma b = -2.16$, a measure of the mean width of the line. Taking for γ its value in the case of two quantum states $\beta = 1/2$ the mean width becomes 0.014 A.U. This is less than the Doppler broadening of a line, for which in the case of Ti atoms at 5200°

we find $b = \frac{\lambda}{c} \sqrt{\frac{2\pi\kappa T}{m}} = 0.035 \text{ AU}$. The order of magnitude, however, is right; and then we must bear in mind the large numerical uncertainties in several of the data used. The absolute intensities of the chromospheric radiations have only roughly been determined (cf. *Results* p. 29 and 32); furthermore the computed value of γb depends on the value chosen for h (by a decrease of h the emission computed is increased in the ratio \sqrt{h} and the absorption $\kappa_0\rho_0 h$ is decreased in the same ratio \sqrt{h}), so that by a diminution of h to 100 km γb is increased 5 times. If still unknown blends are included in our line intensities the slope of our curves $I = f(E)$ will be too small, and then γ_0 must have been found too large. Moreover the value found for the vertical optical depth $\kappa_0\rho_0 h = 1.6$ makes it questionable, whether the simple assumptions stated in art. 1 should not be replaced by a second approximation taking account of the mutual radiation of the atoms.

Addendum.

In *B. A. N.* 157 Dr. WOLTJER brings forward against MILNE'S theory of the chromosphere the objection that its brightness according to theory should be much larger than has been found from the spectrograms of the British Sumatra expedition. I think this difficulty can be removed by considering our result on the small width of the chromospheric lines. With the extreme tenuity of the $Ca +$ gas in the chromosphere we may expect the width of the spectral lines to be determined also entirely by the molecular velocities, thus to be nearly 0.04 AU. Then with the suppositions of

Dr. WOLTJER on a large optical depth we find that this chromosphere will emit a K line with intensity $1/20$ of the photospheric intensity over a width of 0.14 AU, thus will emit an amount of energy $1/500$ of the energy contained in 1 AU of the photospheric spectrum at the same wavelength. If we compare this with the result from the English negative $1/250$ of this same value for the intensity in a prominence of $45''$ height, (*M. N.* 88, p. 546), it cannot be said that theory gives too large a result.