Astronomy. — "*Expansion of a cosmic-gassphere, the new stars and the Cepheids*". By Dr. A. PANNEKOEK. (Communicated by Prof. W. DE SITTER).

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I.

The new stars which are most fully known in their changes of intensity are of two distinct types. The sudden quick flaming up is common to both; but they differ in their further changes of light. To the one class belong the two brightest Novae of this century, Nova Persei 1901 and Nova Aquilae 1918, as well as Nova Coronae 1866. Immediately after attaining the greatest brilliance the light begins to decrease quickly; then the diminution becomes slower, while a periodicity sets in. In the other class, of which Nova Aurigae 1892 is the best known example — and to which Tycho's star Nova Cassiopeiae 1572 belongs — the star retains its brilliance for a long time, fluctuates irregularly, and finally loses its brilliance rather rapidly. These two types of change of light show a certain correspondence with the two types of light-maximum, long and short, which are observed, alternately in the same star, in the Antalgol stars such as SS Cygni. Whether this analogy is more than an accidental correspondence, or that a real relationship exists, cannot yet be ascertained.

In connection with the appearance of Nova Aurigae SEELIGER has given an explanation which fits the phenomena of this type very well; when a star enters a nebulous mass, thereby being brought to a high temperature, as long as it flies through denser and thinner parts, its temperature will fluctuate up and down. This theory fits the other type less well. Here there is obviously an enormous rise of temperature, caused by a momentary event, of which all the further processes are merely the consequences.

The cause from which this sudden heating arises need not be discussed here. We only put the question of what may be deduced concerning the further events, from simple hypotheses. A cosmic body, suddenly brought to such a high temperature, will not be in equilibrium. It will expand adiabatically, and as a consequence it will cool down. Usually the loss of heat by radiation is given as the principal cause of the cooling of a star; but the cooling from adiabatic expansion is of much more importance. In a first approximation, therefore, radiation may be neglected. The force of gravity is also left out of account, which to some extent diminishes the force of expansion; this may be done the more legitimately as it is to a greater or less extent compensated by the radiation-pressure. We assume that all changes take place homocentrically.

A volume-element at a distance r_0 from the centre is found by expansion after a time t at a distance $r = r_0 + \Delta$. We must then have the relation

$$\frac{d^2\Delta}{dt^2} = -\frac{1}{\varrho}\frac{dp}{dr}.$$

A volume-element $r_0^2 dr_0 dw$ shifts to the distance r and becomes $r^2\, dr\, dw$. By this the density changes according to:

$$\frac{\varrho}{\varrho_0} = \frac{r_0^2}{r^2 \dfrac{dr}{dr_0}}.$$

As the change takes place adiabatically, $p\varrho^{-1.4}$ remains constant, or $\varrho = \text{Const.} \times p^{5}/_{7}$, therefore:

$$\frac{d^2\Delta}{dt^2} = -\frac{p_0^{5/7}}{\varrho_0} p^{-5/7} \frac{dp}{dr} = -\frac{7}{2}\left(\frac{p_0^{5/7}}{\varrho_0}\right)\frac{dp^{2/7}}{dr}.$$

The *index* 0 specifies the conditions at the time 0, which thus remain a function of r. We shall indicate by the index 00 the condition for $t = 0$ and $r = 0$, at the centre therefore; putting

$$\left(\frac{p}{p_0}\right)^{2/7} = y, \qquad \frac{p_{00}}{\varrho_{00}} = \alpha, \qquad \frac{p_0}{p_{00}}\frac{\varrho_{00}}{\varrho_0} = \beta;$$

we find

$$\frac{d^2\Delta}{dt^2} = -\frac{7}{2}\frac{p_0}{\varrho_0}\frac{dy}{dr}.$$

Then α is a constant for this gas ball, of the dimension L^2T^{-2}: it is according to $p = \varrho HT$ (H = gas-constant) proportional to the temperature at the centre, and has the physical meaning of the square of the speed of propagation of isothermic disturbances of equilibrium at the centre. β is a number without dimensions, which at the centre is $= 1$, a function of r, which gives the course of α from the centre outwards in the initial condition. The equation of motion now becomes:

$$\frac{d^2\Delta}{dt^2} = -\frac{7}{2}\alpha\beta\frac{dy}{dr} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where y is determined by

$$y = \left(\frac{p}{p_0}\right)^{2/7} = \left(\frac{\varrho}{\varrho_0}\right)^{2/5} = \left(\frac{r_0^{\,2}}{r^2\,\dfrac{dr}{dr_0}}\right)^{2/5}.$$

or

$$y = \left\{\left(\frac{r}{r_0}\right)^2 \frac{dr}{dr_0}\right\}^{-2/5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

y gives the change of the temperature.

By the formulae (1) and (2) the change of Δ with the time is determined, when the quantity β, which determines the original condition, is known as a function of r. We may, for instance, assume a density distribution such as EMDEN has calculated for a gaseous sphere in equilibrium, but supposing a much higher temperature at each point than belongs to an equilibrium form of this kind. The original conditions must be such that Δ continually increases, owing to the strong force of expansion which is working all the time towards the outside in consequence of the high temperature. This will cause the temperature of each layer to fall in a ratio which is given by the quantity y, and in consequence the luminosity will decrease. The most external coldest layers, which absorb the light of the central parts, will move towards us with great rapidity; *this explains why in all new stars, as soon as the light begins to decrease, the dark absorption-lines are displaced strongly towards the violet* — a phenomenon which it has been attempted in vain to explain by a rapid approach of the whole star, or by differences of pressure.

Even when the initial conditions are simple, the equations (1) and (2) are difficult to integrate. An attempt to find the course of the change by mechanical quadrature failed through the fact that small variations in y come out greatly increased in $\frac{dy}{dr}$, and therefore also in the Δ that is found and the subsequent values for y, so that each step gives an increasing inaccuracy, which, after integration through a few units of time, makes the results quite unreliable. On this account we have not succeeded in explaining the periodic variation in brightness — which both in Nova Persei and Nova Aquilae began to appear after the star had decreased 4 classes of magnitude — by special initial conditions.

On the other hand the general mean course of the process may be calculated. The question may be asked: is it possible for the whole mass of the star to expand completely uniformly and what are the initial conditions required in this case?

We must then have $r = r_0 f(t)$, where f is independent of r. It follows that

$$\frac{\varrho}{\varrho_0} = f^{-3} \quad ; \quad \frac{p}{p_0} = \left(\frac{\varrho}{\varrho_0}\right)^{7/5} = f^{-21/5} \quad ; \quad \frac{T}{T_0} = \left(\frac{\varrho}{\varrho_0}\right)^{2/5} = f^{-6/5}$$

$$\frac{1}{\varrho}\frac{dp}{dr} = \frac{1}{\varrho_0}\frac{dp_0}{dr_0} f^{-11/5} \qquad \frac{d^2r}{dt^2} = r_0 \frac{d^2f}{dt^2}$$

hence the equation of motion (1) becomes

$$r_0 f'' = -\frac{1}{\varrho_0}\frac{dp_0}{dr_0} f^{-11/5}$$

For $t = 0$ this gives

$$r_0 f_0'' = -\frac{1}{\varrho_0}\frac{dp_0}{dr_0}$$

The quantity f_0'' must be a constant; a special distribution of density and pressure along the radius for $t = 0$ must therefore be sought, in which

$$-\frac{1}{r_0\,\varrho_0}\frac{dp_0}{dr_0} = A^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(The dimension of A is T^{-1}); f is then determined by

$$f'' = A^2 f^{-11/5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By integration of this equation f is found as a function of the time

$$\tfrac{1}{2}\left(\frac{df}{dt}\right)^2 = -\frac{5}{6} A^2 f^{-6/5} + Const.$$

The constant is determined by the condition that for $t = 0$ and $f = 1$, when the expansion begins, its velocity is 0, hence

$$\left(\frac{df}{dt}\right)^2 = \frac{5}{3} A^2 (1 - f^{-6/5})$$

$$\frac{df}{dt} = A\sqrt{\frac{5}{3}}\sqrt{1-f^{-6/5}}$$

$$A\,dt = \sqrt{\frac{3}{5}}\frac{df}{\sqrt{1-f^{-6/5}}}$$

This form can be integrated by partial integration in the form of a series. Putting

$$f^{-6/5} = x.$$

we have

$$\int x^{-11/6}(1-x)^{-1/2}dx = \frac{6}{5}x^{-5/6}(1-x)^{1/2}\left\{1 - \frac{2}{1}x - \frac{2.4}{1.7}x^2 - \frac{2.4.10}{1.7.13}x^3 - \ldots\right\} + const.$$

or

$$= 2x^{-5/6}(1-x)^{1/2}\left\{1-\frac{2}{9}(1-x)-\frac{2.4}{9.15}(1-x)^2-\frac{2.4.10}{9.15.21}(1-x)^3-\ldots\right\} + const.$$

whence

$$At = \sqrt{\tfrac{3}{5}}f\sqrt{1-f^{-6/5}}\left\{1-\frac{2}{1}x-\frac{2.4}{1.7}x^2-\frac{2.4.10}{1.7.13}x^3-\ldots\right\} + 1.881$$

or

$$= \sqrt{\tfrac{5}{3}}f\sqrt{1-f^{-6/5}}\left\{1-\frac{2}{9}(1-x)-\frac{2.4}{9.15}(1-x)^2-\frac{2.4.10}{9.15.21}(1-x)^3-\ldots\right\} \quad (5)$$

The former series may be used when x is small, or f very large, the latter holds for small f, x being near unity. The additive constant in the second series must be equal to 0, since for $t = 0$ x and f are both equal to 1. For the first series the constant cannot be determined by this condition: it was found by computing the value of At for one and the same value of x from both series.

By means of these series the time-function At was calculated for a number of values of the expansion-factor f. The temperature is connected with f by the relation

$$T = T_0 f^{-6/5}$$

Since the temperature changes according to this law throughout the entire star, we are entitled to assume that the same law holds for the effective temperature; the radiation per unit area then changes proportionally to T^4, i. e. to $f^{-24/5}$. The surface itself changes as f^2, and thus the luminosity as $f^{-14/5}$, hence

$$\log L - \log L_0 = -{}^{14}/_5 \log f.$$

or expressed in terms of classes of magnitude

$$m - m_0 = 7 \log f. \quad (6)$$

The following table contains for a number of values of f the corresponding At and $m - m_0$

f	At	$m - m_0$
1.0	0.0	0.0
1.1	0.455	0.290
1.2	0.655	0.554
1.3	0.815	0.798
1.4	0.956	1.023
1.5	1.086	1.233
1.6	1.207	1.429
1.7	1.322	1.613
1.8	1.433	1.787
1.9	1.541	1.951
2.0	1.645	2.107

t	At	$m - m_0$
2.5	2.137	2.786
	2.135	
3.	2.599	3.340
4.	3.479	4.214
5.	4.327	4.893
10	8.397	7.000
20	16.307	9.107[1])

The intensity thus diminishes slowly at first and then faster and faster, but the velocity soon reaches a maximum, when the star has fallen rather more than 1 magnitude below the original intensity. The velocity of decrease then becomes slower once more and finally approaches a logarithmic curve.

The slow decrease in the beginning is not observed in the new stars, as the process of blazing up has not yet worked out then. Both Nova Persei and Nova Aquilae had their maximum one day after they had reached the first magnitude, and Nova Persei one day before that had already attained the 3rd magnitude. As the starting-point $t = 0$ we must not therefore take the moment of maximum luminosity, but one or two days earlier. Then follows a rapid decrease which, however, soon becomes slower and is then accompanied by periodical variations. On comparing the observed light-curve and the one here calculated they are found not to agree during the further course of the change; the mean observed intensity decreases much more slowly than according to the above calculation. Evidently other influences are at work here, lying outside the simple theory here given. It is therefore only for the first period of rapid decrease of luminosity that agreement may be looked for.

For Nova Persei we shall take 0,0 as the ideal maximum intensity, a little higher than the greatest brightness observed, because for it the final stage of the blazing up overlaps the beginning of the expansion, and for the ideal starting-point the 21st of February. The following values of At are then found from the values of $m - m_0$ on the smoothed observational light-curve.

Date	m obs.	At	t	quotient	A
Febr. 25	1.00	0,97	4^d	0.24	$2{,}8.10^{-6}$
„ 27	1.62	1.35	6	0.22	2,6 „
Mrch 1	2.07	1.62	8	0.20	2,3 „

[1]) From 1.0 to 2.5 equation 5*b* has been used, from 2.5 to 20 equation 5*a*.

Date	m obs.	At	t	quotient	A
Mrch 3	2.42	1.85	10	0.18	$2{,}2.10^{-6}$
„ 5	2.73	2.07	12	0.17	2,0 „
„ 7	3.02	2.30	14	0.16	1,9 „
„ 9	3.27	2.53	16	0.16	1,8 „
„ 11	3.48	2.71	18	0.15	1,7 „
„ 13	3.65	2.88	20	0.14	1,7 „

The diminution of the quotient shows that those influences which later on retard the decrease to a higher degree than the theory requires, begin to manifest themselves even in the first stage. However that may be, the order of magnitude of A as found here cannot but be correct, and from it conclusions may be drawn as to the constitution of the Novae.

When all quantities are expressed in the absolute system, t is measured in seconds; taking 0.21 as a mean value of the quotient in the above table we have

$$A = 0{,}21 : 86400 = 2{,}5 \,.\, 10^{-6}.$$

For Nova Aquilae about the same value is found.

II.

The distribution of pressure and temperature for $t = 0$, which is required for a uniform expansion, and the dependence of A on this distribution are determined by equation (3) in which we shall now leave out the indices 0:

$$-\frac{1}{r\varrho}\frac{dp}{dr} = A^2$$

or

$$\frac{dp}{dr} = -A^2 r\varrho$$

or

$$p = +A^2 \int_R^r \varrho r \, dr \, .$$

where R is the radius of the external surface, where $p = 0$. If a definite density-distribution is assumed as existing at the moment of flaring up, the latter equation determines the pressure as a function of r, and therefore also the temperature:

$$T = \frac{A^2}{H\varrho}\int_R^r \varrho r \, dr. \quad \quad (7)$$

For the density the values have been assumed which EMDEN has calculated for the equilibrium-forms of spherical cosmic gaseous masses (for $n = 2^1/_2$, $k = 1.4$); the integration has been performed by mechanical quadrature. The integration-intervals were taken four times smaller than the unit of r_1 as used by EMDEN; expressed in our unit the radius of the external surface is 21.67. The result of the integration was as follows:

r	$log\ \varrho$	$21.67^2 \int_R^r \frac{\varrho}{\varrho_0} \frac{r\,dr}{R^2}$	$I = \frac{21.67^2\, \varrho_0}{\varrho} \int_R^r \frac{\varrho}{\varrho_0} \frac{r\,dr}{R^2}$
0	0.00000	22,5274	22.53
2	9.95523	20,6746	22.92
4	9.82604	16.0941	24.02
6	9.62544	10.8138	25.62
8	9.36944	6.3898	27.30
10	9.06954	3.3527	28:56
12	8.72945	1.5525	28.94
14	8.34375	0.6148	26.61
16	7.88832	0.1922	24.84
18	7.29284	0.03928	20.04
20	6.32566	0.00315	15.12

The integral I is proportional to the temperature. The result therefore shows, that *the uniform expansion requires a distribution of temperature which differs very little from an even temperature throughout the mass.* If the original process is not a rise of temperature at the surface by friction in a nebulous mass, but if through some catastrophe the entire mass becomes hot throughout, an approximately equal temperature through the whole mass might be expected and in that case, as was here shown, an approximately uniform expansion would take place.

Now for

$$I = \frac{21\,67\, \varrho_0}{\varrho} \int_R^r \frac{\varrho}{\varrho_0} \frac{r\,dr}{R^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

we have

$$T = \frac{IR^2 A^2}{21{,}67^2.H} = \frac{IR^2A^2\mu}{21{,}67^2.8{,}3.10^7}$$

if μ is the molecular weight of the gas of which the star consists. Substituting the value of A found above, the mean temperature (taking $I = 25$) becomes:

$$T = \frac{25 \times 6.25.10^{-12}}{21{,}67^2 \times 8{,}3.10^7} R^2\mu = 4.10^{-21} R^2\mu \quad . \quad . \quad . \quad . \quad (9)$$

With $T = 10^4$ degrees this gives:

$R^2\mu = 2,5.10^{24}$ hence for $\mu = 1$ (dissoc. H) $\mu = 50$ (metals)

$R = 1,6 \times 10^{12}$ $2,3 \times 10^{11}$

$= 23$ times the sun 3,5 times the sun

and with $T = 10^5$ degrees

$R^2\mu = 2,5.10^{25}$ hence $R = 5 \times 10^{12}$ 7×10^{11}

$= 71$ times the sun 10 times the sun

Considering that at this high degree of heat the mass will be highly dissociated, the first values are probably nearer the truth than those corresponding to $\mu = 50$. It shows that *a Nova at the moment of greatest brightness is a body much more gigantic than the sun, not only in luminosity but also in radius and volume.* The theory, that a new star arises when a dark body of the size of our sun, i.e. an ordinary cooled-down dwarf star, suddenly rises to a colossal temperature, is in contradiction with the above calculations; for $R = 7 \times 10^{10}$, the radius of our sun, with $\mu = 50$ and A as observed, the temperature would only rise to 1000°; for $T = 10^5$ A would be 10 times larger, that is: the time in which the star loses its light would be 10 times smaller.

This result is in accordance with the value of 0″.011 for the parallax of Nova Persei, derived by KAPTEYN from the supposition that the nebulous rings which were photographed half a year later arose from reflected star-light. This leads to a luminosity 10000 times that of the sun; since the intensity of the surface-radiation was not much different from what it is in an ordinary white star — HERTZSPRUNG found a similar distribution of light in the spectrum of Nova Aquilae as in α Aquilae [1]) — the radius of the Nova must have been 30 to 50 times the radius of the sun.

Supposing our interpretation of the dark lines which always accompany the bright lines on the violet side being correct, this also leads us to a high value of R. The velocity with which the outermost particles move towards us is $R^{dJ}/_{dt}$. At the moment when the light has fallen by two magnitudes, we have $^{df}/_{Adt} = 1$, hence $R^{dJ}/_{dt} = AR = 2.5 \times 10^{-6} R$. For $R =$ the radius of the sun this would become 1.7 km. per second. On the other hand the observed displacement of the dark lines was as much as would correspond to 700 km. per second. The real velocity must have been smaller, however, since the absorption-line is partly effaced by the broad adjoining emission-line; on the assumption that the velocity may have been about 100 or 200 km./sec. R is found equal to 60 or

1) Astronomische Nachrichten Bd. 207. Nr. 4950.

120 times the radius of the sun, therefore again a value of the same order of magnitude.

The Novae in the first stage of their brightness thus possess the characteristics of the giant-stars; in order that their mass may not become too exceptionally large, their density must be small even before the expansion. The relation found here between T, R, and A cannot teach us anything on this point, as it does not contain the density. A further indication for a small density may be found, however, in the fact that after a decrease of 4 magnitudes the spectrum at the minima of the light-variations more and more approached the character of a nebula-spectrum, and after another few months the star had become a nebula. At this stage the density has become so small that the visible emission is derived from the whole body including even the hindmost layers and still gives but a feeble surface-brightness; the fact that this condition sets in, when the expansion factor has become something like 10 or 20, proves that the original density must also have been far below unity.

III.

The original equation of motion (1) may also be written in such a form that it does not contain any dimensions.

Let us put

$$r = \gamma s \qquad \Delta = \gamma x \qquad t = \delta z \quad . \; . \; . \; . \; (10)$$

where γ is a linear measure, δ a length of time and s, x, and z are numerical values. The equations then become

$$\left.\begin{array}{ll} \dfrac{d^2x}{dz^2} = -\dfrac{7}{2}\alpha\beta\dfrac{\delta^2}{\gamma^2}\dfrac{dy}{ds} = -B\dfrac{dy}{ds} & \\ s = s_0 + x\,; & y = \left(\left(\dfrac{s}{s_0}\right)^2\dfrac{ds}{ds_0}\right)^{2/5} \end{array}\right\} \quad . \; . \; . \; . \; (11)$$

where β is a function of the coördinate s_0. The function β and the constants α, γ, δ which determine the special constitution and size of the star are united in the one coëfficient B. The law of change of x with z is solely dependent on this coëfficient, and is the same for all bodies with the same B. Equations (11) determine all possible movements — progressive, irregular or periodical, which may occur in a cosmic gaseous mass, in so far as they are a function of r only and as gravitation may be left out of account. Without calculating these movements themselves, a relation of similarity may be derived from the formulae which establishes a connection between the changes in different stars. If for different

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cosmic gas-spheres the distribution of β along the radius is the same, the expression

$$x = \frac{\Delta}{\gamma} = f\left(\frac{r}{\gamma}, \frac{t}{\delta}\right)$$

must be the same function for them, provided B i. e. $\alpha\beta\frac{\delta^2}{\gamma^2}$ for them is the same number. If for each of them a suitable time- and distance-scale is assumed, the motions and variations expressed on this scale are for all these bodies identical.

Assuming that a periodical solution of the equations (11) exists in which the particles move radially to and fro and the density periodically becomes adiabatically larger and smaller, this condition of motion will be valid for all such bodies provided the periods of the variations are expressed in δ as unit and the dimensions of the bodies in terms of γ. We must then have the relation

$$\alpha_1 \delta_1^2 \gamma_1^{-2} = \alpha_2 \delta_2^2 \gamma_2^{-2}$$

Now $\alpha = HT_{00}$ (at the centre), therefore proportional to the temperature at the centre. Calling P the period of the variations and R the radius of the gas-sphere, this gives:

$$P_1^2 : P_2^2 = \frac{R_1^2}{T_1} : \frac{R_2^2}{T_2} .$$

If we may assume, that similar bodies of this kind have the same temperature, the brightness becomes proportional to R^2, i. e. to P^2. Otherwise the temperature will still depend on some power of R and we have the more general relation

$$P^2 \sim L^n$$

or

$$2 \log P = Const. + n \log L$$

or

$$2 \log P = Const. - 0{,}4\, n \times M$$

if M represents the absolute magnitude. A relation of that kind was found by Miss LEAVITT for the variable stars of the δ Cephei-type in the small Magelhanic-cloud [1]). For 25 stars with periods from 1.25 to 127 days she found, that the period increased with the magnitude in such a manner that the logarithm of the period changed by 0.48 per magnitude-class.

The Cepheids are giant-stars, to which our suppositions are in so far applicable, that gravity, small in itself by the small density, must moreover for the greater part be neutralized by the radiation-

[1]) Harvard Circular Nr. 173.

pressure. They are all nearly of the same spectral type, hence their temperature cannot differ much. The relation which has been found to hold for them between period and intensity may therefore be explained in a simple manner by assuming that the variation of light arises from a pulsation of the gaseous sphere; not, as is often assumed, a pulsating deformation, but a pulsating expansion and contraction. Hereby the absorbing layers at the front of the star will alternately move away from us and towards us, hence in the spectrum a periodical displacement will take place. This displacement has usually been taken as indicating an orbital movement and for this reason the Cepheids are admitted amongst the spectroscopic double stars. Still amongst these they occupy a very exceptional position. Calculating the mass from the elements of the orbit, very much smaller values are found for the Cepheids than for other spectroscopic double stars, although their volume is much larger than that of the sun. Although an extremely small density is not altogether impossible a priori, still in the relatively small radial velocity an indication may be seen for the assumption, that a different explanation must be given here than for ordinary spectroscopic double stars.

But the question arises: is it possible that from an expansion and contraction a radial velocity arises of such a value as the experiments give — of several times ten kilometers per second?

The luminosity of δ Cephei and η Aquilae was found by ADAMS from the spectrum to be 60 times that of the sun; for a mean Cepheid with a period of 6 6 days HERTZSPRUNG derived from the proper motions 600 times the luminosity of the sun. Assuming on the ground of the accordance as to spectral type and colour an equal radiating power per unit surface, these results give a radius equal to 8 and 24 times respectively that of the sun. Representing the maximum expansion and contraction by the factor $f = 1 + \Delta f$, the maximum radial velocity will be

$$V = \frac{2\pi \Delta f . R}{86400 P},$$

where P is the period in days and R the radius. In kilometres R is 8 or $24 \times 7 \times 10^5$. Taking for P 6 days, this gives

$$V = \frac{6{,}3 \Delta f \times 8 \text{ (of } 24) \times 7.10^5}{4{,}3 \times 10^5} = 82 \text{ resp. } 246 \Delta f \text{ KM.}$$

Since these Cepheids fluctuate rather less than 1 magnitude visually and rather over 1 magnitude in photographic intensity, we shall assume one magnitude for the variation in complete radiation;

therefore *log* L varies by the amount 0.20 above and below the mean. If the radius changes as the number f, the density changes as f^3, the temperature as $f^{6/5}$ and the radiation as $f^{24/5}$; from $\log L = \pm 0.20$ it then follows that $\log f = \pm 0.04$, hence f fluctuates between 1,1 and 0,9. In the expression for V we must therefore take 0,1 for Δf and the maximum radial velocity becomes 8 or 25 kilometers per second. This value has to be somewhat lowered, since spectographically the mean velocity of the entire front surface is measured, of which only the central parts have the velocity which we have here calculated. But even then the value found agrees sufficiently with the measured velocities (10 to 20 kilometers per second) to admit the explanation of the light variation and the variation in radial velocity on the ground of contraction and expansion.

There are some other objections to this explanation. The one is the same objection which also holds against the explanation through an orbital movement viz. that the maximum intensity coincides with the highest velocity towards us. The other objection lies in the coefficient 0.48 found by Miss LEAVITT. If for these Cepheids equality of spectral class and thus of emissive power and of T may be assumed, the brightness becomes proportional to the surface, which gives

$$P^2 \sim L.$$

or

$$\log P = Const. - 0{,}2\,M.$$

In this case therefore the coëfficient should be 0.2, whereas Miss LEAVITT finds a much larger change of the period or a much smaller change of the brightness. It is therefore difficult to explain the deviation by means of a dependence of the temperature T on the linear dimension R; for in that case T would have to be smaller, the larger the star. Possibly an explanation may be found by assuming, that the mass of the Cepheids is actually small, and therefore the density very low, so low, that the rays emitted from one side of the star may penetrate the complete body without being completely absorbed. If a glowing gas-sphere is so rare, that we observe the emission even from the hindmost layers without any diminution, the total light from the sphere will no longer be proportional to its surface, but to its mass, therefore be the same for two bodies of equal mass and different dimension. Intermediate conditions are conceivable in which the total light will then be proportional to a lower power of R, say to the first power. In the latter case the coëffficient of M in the formula for $\log P$ would become about 0.40.
