Astronomy. — *"The variability of the Pole-star."* By Dr. A. PANNEKOEK. (Communicated by Prof. E. F. VAN DE SANDE BAKHUYZEN).

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A slight variability of α Ursae minoris has already several times been suspected by different observers (SEIDEL, SCHMIDT). When in 1889 and 1890 I executed a great number of observations (estimates with the naked eye after ARGELANDER's method) for the determination of the brightness of the stars of the 2[nd] and 3[d] magnitudes, such great differences showed in some of these stars, that they were being observed as regularly and as often as possible in the following years with a view to probable variability. Among these stars was also the Pole-star [1]). In 1890 I found that the period was about 4 days; each time 2 days after a great intensity came a faint one and the reverse. I did not succeed, however, in finding an accurate value for the period. From the observations in December 1890 I found two maxima on Dec. 7.0 and Dec. 29.8 (in reality they occurred on Dec. 6.6 and Dec. 30.4), which yielded a probable period of 3.8 days; this however did not agree with the observations of that winter.

After all it must indeed have been hopeless to derive the elements of the variation from these observations only. As the mean error of an estimate amounted to 0.7 of the whole amplitude, as appeared later on, it might even happen that a maximum and a minimum seemed to have changed places owing to errors of observations. Moreover the remembrance of the results of previous days may spoil an observation. If on one particular day the star has, perhaps wrongly, been estimated very faint, one expects to see it very bright two days afterwards, and this may influence the estimate. On the other hand the small number of observations in a given interval of time, say a month, owing to bad weather, did not allow to counteract the uncertainty of the separate estimates, by uniting a great number into a normal place. I have long continued the observations of this star, up to 1899, in order to have material for a closer investigation, in case the variability should be proved and the period should be accurately known.

In 1898 CAMPBELL discovered that the radial velocity of this star is variable and hence that it is a spectrocopic double star with a

1). The other stars in which I consider variability to be probable, although I cannot prove it with certainty owing to the smallness of the amplitude, are ζ Tauri (period of a few days), 40 Lyncis (26 days) and π Herculis (14 months); the latter two are of a red colour.

period of 3.968 days. Lack of time, because of my work at the observatory, prevented me from immediately reducing my observations by means of this value for the period and so testing the variability. The probability that α Ursae minoris was indeed a short-period-variable of the type of δ Cephei grew stronger, when I found in 1906 [1]) that it showed the same peculiarity in its spectrum as those stars (c-character after Miss MAURY) and has, as all stars of short period of this type, an extraordinary slight density. In a footnote attention was already drawn to these moments of probability.

Starting from the consideration, that for all these short-period-variables the photographic amplitude is much larger than the visual one, HERTZSPRUNG at Potsdam has thereupon (in 1910 and 1911) taken a great number of photographs (418 plates in 50 nights) of Polaris, and from this settled with absolute certainty a variability with an amplitude of 0.17 magnitude [2]). For the epoch of maximum light he found J. D. 2418985.86 ± 0.08 Greenwich M. T. Subsequently J. STEBBINS has executed a number of photometric measurements with his exceedingly sensitive selenium-method in 1911—12; these also clearly show a variability with a visual amplitude of 0.07 magnitude [3]). The epoch of greatest brighness as found by him, viz. J. D. 2418985.94 Gr. M. T. agrees very well with HERTZSPRUNG's result.

I have also reduced my observations of 1890—1900 with the aid of the periodic time $3^d.9681$, as spectrographically found. In the second half of each year I used for comparison the stars of Perseus and Andromeda, in the first half those of Ursa major. Thus the observations form two mutually independent series, partially overlapping in wintertime. For the 1st series α Persei $= 6.3$, β Andromedae $= 3.8$, γ Andromedae $= 3.1$, and exceptionally α Arietis $= 5.4$ and α Andromedae $= 2.3$ were used as a scale of comparison-stars; for the 2nd series served ε Ursae maj. $= 2.4$, η Ursae maj. $= 0.0$, and, exceptionally, α Ursae maj. $= 4.0$. The observations were not corrected for atmospheric extinction, since this influence disappears in the mean of many observations and at the most can make the mean error seem too great. Taking all together, from 1890 up to 1899 259 comparisons with the Perseus-Andromeda-stars were available and 251 comparisons with those of Ursa major. With the aid of the periodic time 3.968 all epochs of observation were reduced to

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1) See A. PANNEKOEK. The luminosity of stars of different type of spectrum. Proceedings Acad. Amsterdam 9, 1906, p. 134.

2) Astronomische Nachrichten 4518 (Bd. 189, 89).

3) Astronomische Nachrichten 4596 (Bd. 192, S. 189).

one single period, viz. Aug. 3—7 1894, and subsequently united into normal places. These normal places are the following:

First series		Obs.—Calc.	Second series		Obs.—Calc.
Aug. 3.12	3.72 (18)	+ 0.03	Aug. 3.21	0.59 (18)	— 0.03
3.42	3.94 (16)	+ 11	3.48	0.68 (16)	— 08
3.73	4.11 (21)	+ 08	3.65	0.59 (13)	— 28
3.96	3.91 (17)	— 29	3.92	1.13 (18)	+ 07
4.22	3.94 (20)	— 42	4.25	1.39 (20)	+ 10
4.54	4.68 (17)	+ 18	4.46	1.40 (22)	00
4.76	4.86 (16)	+ 33	4.65	1.69 (24)	+ 22
4.94	4.76 (16)	+ 24	4.94	1.55 (14)	+ 05
5.18	4.11 (17)	— 34	5.22	1.20 (17)	— 23
5.52	4.45 (14)	+ 19	5.46	1.27 (18)	— 03
5.76	4.11 (20)	+ 02	5.72	1.02 (20)	— 11
5.94	3.67 (18)	— 29	6.22	0.74 (19)	— 04
6.23	3.67 (16)	— 11	6.48	0.86 (16)	+ 22
6.46	3.78 (14)	+ 10	6.86	0.67 (16)	+ 11
6.73	3.79 (19)	+ 16			

Both series show, as does the graphic representation, with unmistakable certainty a periodical variation of the brightness to an amount of about one scale-unit with a maximum on 4.8 August. The calculation of a sine-formula resulted in (zero epoch 3.0 August):

1st series $4.08 + 0.45 \sin(\varphi - 72°0)$ Maximum 4.79 Aug. $\pm 0^d.13$

2nd series $1.03 + 0.47 \sin(\varphi - 78°9)$ Maximum 4.86 Aug. $\pm 0^d.09$

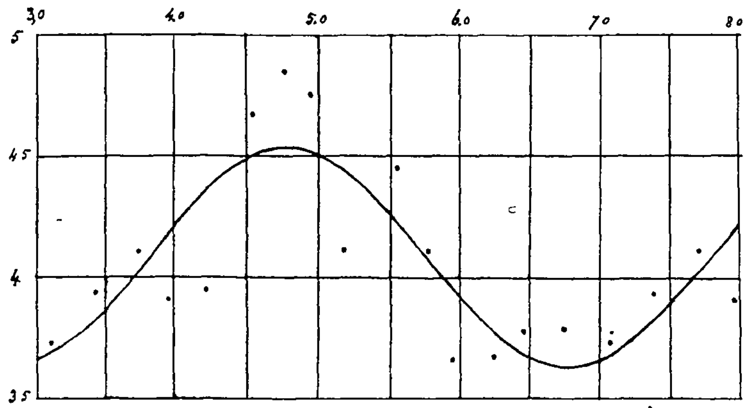

Fig. 1.

The remaining deviations Obs.—Calc. have been placed in the last column. They yield for the mean error of a normal place

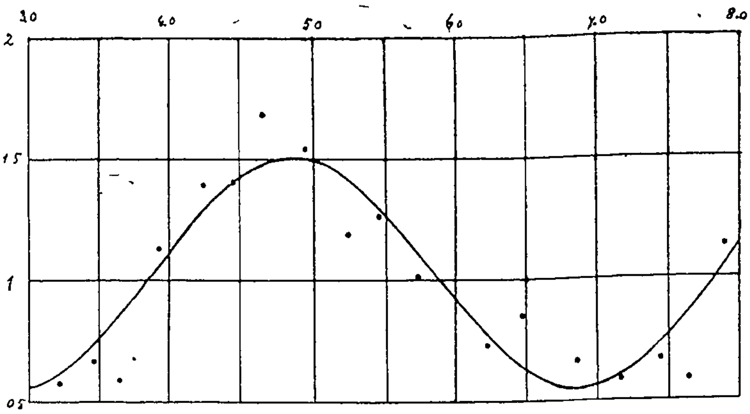

Fig. 2.

according to the mean of the two series, 0.21 (if we adopt this same value for both series, then each maximum has a mean error of $0^d.11$), from which we find 0.84 as mean error of one observation, while 0.7 had been found from the differences between the separate results and the adopted normal places. The deviations of the normal places from the sinusoid, it is true, show a systematic character, in the sense that the maximum is very sharp, the minimum very flat, hence that a term with 2φ is indicated, the positive maximum of which falls together with the maximum of the principal term. Since, however, nothing of this kind is to be observed in the light-curves of HERTZSPRUNG and STEBBINS, no further attention has been paid to this phenomenon. Thus my observations yield as epoch of the maximum, after reduction to Greenwich-time·

1894 Aug. 4.81 Gr. M.T. = J.D. 2413045.81 ± $0^d.08$.

The interval between my normal-epoch and that of HERTZSPRUNG J. D. 2418985 86 is 5940.05 days = 1497 periods of 3.9680 days.

In order to reduce the brightness of maximum and minimum to the same photometric scale, the catalogues of Potsdam and Harvard were used. For the reduction of the magnitudes given there to the homogeneous scale that has been derived and adopted in my dissertation "Untersuchungen uber den Lichtwechsel Algols" (p. 146—158) first a correction was added to the values of Harvard **44**, in order to reduce them to Harvard **14**. This was derived from the differences between the two catalogues, calculated by MULLER and KEMPF and communicated in their "Generalkatalog der photometrischen Durchmusterung" [1]), Einleitung S. XXIII. For our purpose they were given the following form:

$$H. 44 - H. 14 = -0.01 + a (c - 4.0)$$

in which c is the colour-number according to OSTHOFF and a a function of the magnitude, varying linearly with the difference between the apparent brightness of the star in the two photometers, calculated in the manner as has been indicated on p. XXIV of the same introduction (for magnitude 1.0, 2.0, 3.0 we have $a = +0.062$, $+0.054$, $+0.042$). Subsequently to these magnitudes, reduced to H. **14** and to the magnitudes of H. **14** itself, the correction for colour was added, which has been found in my dissertation p. 158. There is also to be found the correction varying with the magnitude which has to be added to the results with Photometer C II, in order to reduce them to the same system [2]. All stars used by me have been observed in Potsdam also with Photometer C III. As they have no excessive apparent brightness in this instrument and hence no variation with the brightness is to be expected in this case, a constant correction $-0^m.23$ was added to the results with C III.

For the employed comparison-stars, supplemented with a few other stars, continuing the scale further to the fainter side, we give successively: the colour according to OSTHOFF, (derived in the manner as indicated in my dissertation p. 168), next the magnitudes of Harvard **14**, Harvard **44**, Potsdam C II and C III, all corrected in the way already mentioned, subsequently the adopted simple mean value from these four and then the brightness in the employed scale of comparison-stars.

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[1]) Publicationen Potsdam **17**.

[2]) MÜLLER and KEMPF have not corrected the results obtained with C II, because they could not discover a systematic difference between C I and C II (Einleitung S. XIV). Since, however, for the comparison of these instruments they could only avail themselves of stars between magnitudes 3.5 and 5.5, this does not clash with my result that a correction is needed for the brighter stars up to the 2nd magnitude, which of course can only be found by comparison with another catalogue. While the comparisons employed by MÜLLER and KEMPF can teach nothing about the absence of systematic errors for these bright stars, the fact that increasing negative corrections are needed for C I above magnitude 4.8, and for photometer *D* above magnitude 6 1 (Einleitung S. XII), renders it exceedingly probable that similar corrections are needed for C II above magnitude 3.5, such as I derived in my dissertation. The final values of the Potsdam "General Catalog" are therefore likely to be systematically erroneous above the 3rd magnitude. For this reason I have not been able to use simply the Potsdam system for the magnitudes of the comparison-stars, as would have been a matter of course for fainter stars. By using the Potsdam system I should have found the amplitude too small.

Star	Colour	H 14	H 44	P.C II	P.C III	Mean	Scale	Calc.
α Persei	3.4	1.94	1.88	1.87	1.95	1.91	6.3	1.92
γ Arietis	5.4	2.03	2 10	—	1.96	2.03	5.4	2.02
β Andromedae	6.2	2.20	2 05	2.04	2.11	2.10	3.8	2.14
γ Andromedae	5.2	2.13	2.19	2.09	2.14	2.14	3.1	2.16
α Andromedae	1.8	2.09	2.22	2.21	2.17	2.17	2.3	2.15
γ Cassiopeiae	2.1	2.32	2.35	2.24	2.23	2.28	0.8	2.25
β Cassiopeiae	2.9	2.43	2.50	2.36	2.33	2.41	—1.7	2.43
α Ursae maj.	4.9	1.96	1.88	1.79	1.77	1.85	4.0	1.86
ε Ursae maj.	1.8	1.86	1.89	1.98	1.84	1.89	2.4	1.89
η Ursae maj.	1.4	2.03	2.03	1.98	2.05	2.02	0.0	2.03
ζ Ursae maj.	2.1	(2.40)	2.29	2.18	2.12	2.20	—3.6	2.25
α Coronae	1.8	2.39	2.38	2.32	2.39	2.37	—4.8	2.32
ε Bootis	4.8	2.55	2.57	2.37	2.52	2.50	—5.7	2.43
β Ursae maj.	1.7	2.63	2.71	2.41	2.42	2.54	—8.9	2.56
γ Ursae maj.	1.8	2.59	2.66	2.54	2.39	2.55	—9.7	2.61

The relations between the scale-values n and the magnitudes m are represented by the following formulae (3.7 is the colour-number of α Ursae minoris):

1st series $m = 2.335 - 0.065\, n + 0.020\, (c - 3.7)$.

2nd series $m = 2.07 \quad - 0.059\, n + 0.020\, (c - 3.7)$.

The magnitudes of the stars calculated after these formulae are given in the last column of the preceding table. With the aid of the same relations the sine-formulae for the brightness of α Ursae minoris, become expressed in magnitudes:

1st series $2^m07 - 0^m029\, sin\, (\varphi - 72°0)$

2nd series $2^m01 - 0^m028\, sin\, (\varphi - 78°9)$.

So the amplitude of the variation of light amounts to 0^m057, while we find as mean error of an observation based on the deviations of the separate observations 0.043 and on the deviations of the normal places from the formulae 0.051.

II.

Among the older material that may serve for the examination of the variability of Polaris, we must in the first place consider the observations executed by G. MÜLLER in 1878—81 at Potsdam for the determination of the atmospheric extinction and published in Vol. III of the Potsdam "Publicationen". As these observations consist in measurements of the differences in brightness between Polaris and 5 other stars observed in very different zenithdistances, they yield abundant material for the determination of the variability of Polaris

For this purpose I have examined the deviations of these differences from their mean value, remaining after correction for mean extinction, which are to be found in MÜLLER's Table IV, last column but one (p. 261—265). Excluded were all observations in which the zenithdistance exceeded 60° and all those indicated as uncertain by the observer. The others were arranged according to the phase, counted from 1879 December 12.0 $+ n \times 3^d 968$. The unit of these deviations is that of the third decimal place of the logarithm of the proportion star: Polaris, i. e. 0.0025 magnitude. In order to give the positive sign to the maximum light, the signs must be reversed. In the following table are given the normal places formed from these deviations reversed in sign and reduced to magnitudes; the number of observations on which each normal deviation depends has been added in brackets.

Epoch	Deviation	O—C	Epoch	Deviation	O—C
Dec. 12.02	$+0^m022$ (25)	$+0^m001$	Dec. 14.11	-0^m028 (24)	-0^m012
12.34	+ 030 (18)	000	14.50	— 009 (20)	+ 015
12.64	+ 047 (23)	+ 014	14.81	— 009 (23)	+ 013
12.92	+ 008 (20)	— 021	15.05	— 021 (25)	— 004
13.21	+ 028 (18)	+ 008	15.20	— 048 (19)	— 036
13.63	+ 006 (20)	+ 003	15.34	+ 017 (24)	+ 022
13.84	— 010 (17)	— 004	15.69	+ 008 (29)	— 001

Here also the variability of Polaris appears with unmistakable clearness and it may be expressed by the following sine-formula:

$$\text{Deviation} = + 0^m004 + 0^m028 \, sin \, (\varphi + 35^\circ)$$

Maximum Dec. 12.61 = 1879 Dec. 12.57 ± 0.14 M. T. Greenwich

The last column of the table contains the differences Obs.—Calc. The mean error of a mean value from about 22 observations is 0^m016, hence the mean error of one observation 0^m077.

The immense number of photometric measurements made at the Harvard Observatory, in which Polaris has been used as comparison-star, have already been condensed into normal values by PICKERING [1]). Calculating the time of maximum light also from the mean deviations given by him, by means of a sine-formula, we obtain:

$$\text{Deviation} = +0^m02 + 0^m039 \sin(\varphi + 254^\circ)$$

Phase	Deviation		O.—C.	Phase	Deviation		O.—C
0^d2	$+0^m01$	(120)	$+0^m047$	2^d2	$+0^m03$	(123)	-0^m011
0 .6	— 06	(197)	— 031	2 .6	+ 03	(179)	— 002
1 .0	— 02	(152)	— 012	3 .0	00	(168)	— 012
1 .4	— 01	(126)	— 025	3 .4	— 03	(169)	— 018
1 .8	+ 09	(126)	+ 056	3 .8	— 02	(150)	+ 011

The last column again contains the differences Obs.—Calc. The mean error of a normal deviation is 0^m033. As a positive sign here means a greater brightness of Polaris, the maximum-light occurs at the phase $2^d.16 \pm 0^d.24$. The zero epoch of the phase is at J. D. $2400000 + 3.9683\,E$; for $E = 2073$ this becomes J.D. 2408226.29, so that the normal epoch of maximum becomes

J. D. 2408228.45 ± 0.24.

III.

Putting together the hitherto obtained results for the light-variation of α Ursae minoris and comparing them with the formula for the maxima given by HERTZSPRUNG:

$$\text{J. D. } 241\,8985.86 + 3.9681\,E$$

we find the following table:

Year	E.	*Observed* 24	*O—C*	*Amplitude*	*Observer*
1879	— 2845	07696.57 ± 0.14	-0^d05	0^m056 vis.	MÜLLER
1881	— 2711	08228.45 ± 0.24	+ 0.11	0 .078 vis.	HARVARD
1894	— 1497	13045.81 ± 0.08	+ 0.20	0 .057 vis.	PANNEKOEK
1910	0	18985.86 ± 0.08	0.00	0 .171 ph.	HERTZSPRUNG
1911	(+ 100)	18985.94 ± 0.09	+ 0.08	0 .078 sel.	STEBBINS

Attempting to correct with these data HERTZSPRUNG's formula, we find (adopting as weights 2, 1, 4, 4, 4) as correction:

$$+0^d.07\,(\pm 0^d.06) - 0.00001\,(\pm 0.00004)\,E$$

Thus for the length of the period the exact value adopted by HERTZSPRUNG is found. The most probable formula for the maximum-epoch of α Ursae minoris now becomes:

$$\text{J. D. } 241\,8985.93\,(\pm 0.06) + 3.96809\,(\pm 0.00004)\,E.$$

[1]) Harvard Circular Nr. 174, Astronomische Nachrichten 4597 (Bd. 192. S. 219).