Citation:

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EULER accepts the views of DUCLAUX and BLACKMANN and, besides, the said hypothesis. According to our results the said theory must now be rejected: also without destruction the function shows an optimum curve. We have thus to admit that if heating above this optimum were possible without destruction a reversible inactivation of the enzyme would be observed, but there is no cause to speak with EULER of such an inactivation already below that temperature and therewith to explain the deviation from VAN 'T HOFF's rule.

The obtained result also throws light upon other observations: the occurrence of the "Wärmestarre" to explain which the hypothesis of DUCLAUX-BLACKMAN meets with great difficulties, is now easily explained by considering that in cases where this phenomenon takes place, the destruction proceeds extremely slowly so that the optimum curves for different (not too long) times of fore-heating actually fall together with those of 0′ fore-heating.

Moreover, the influence of the temperature proves now to follow the same law as was observed for other influences on physiological processes, by which the process is accelerated to a certain degree, then slackened and finally stopped.

Lastly we bring our best thanks to the Direction of the Dutch Yeast- and Spirit Works here for the readiness with which the yeast for the described experiments was supplied us.

Delft, May 1910.

Astronomy. — "*Researches into the structure of the galaxy.*" By Dr. A. PANNEKOEK. (Communicated by Prof. E. F. VAN DE SANDE BAKHUYZEN).

§ 1. The researches into the structure of the universe propose to ascertain the star-density (quantity per unit of volume) as a function of their place in space, i.e. for any direction the star-density as a function of the distance to the sun. If all the stars had the same absolute luminosity, the apparent brightness m expressed in magnitudes would be a direct measure for the distance r according to the relation $0{,}2\, m = \log r$. The number of stars of a given magnitude $A(m)dm$ would then indicate immediately the density Δ for the corresponding distance r, according to $A(m)dm = \Delta(r) \,.\, r^3 dr$.

The stars, however, are of different luminosity and therefore the number of stars $A(m)$ is in a more intricate way dependent on the density Δ. If the function expressing the number of stars of different luminosity is known, inversely Δ can be concluded from the values of A. For the present we must assume that the function expressing the distribution of stars over different luminosities, is the same everywhere. This function has been determined by KAPTEYN. The values of the logarithm of the number of stars per unit of volume $log\,\psi$ as a function of the logarithm of the luminosity $log\,L$ 1) can be represented by a parabola with a maximum for $log\,L = 8.2$, if for the sun we put $L = 1$. If the luminosity is expressed by the apparent brightness H in magnitudes which the star would show at a distance $\pi = 0''.1$, then we find $H = 5.5 - 2.5\,log\,L$ (the sun having the magnitude 5.5 at this distance), so the maximum lies at $H = 10$ and the function ψ can be expressed thus:

$$log\,\psi = c - 0.025\,(H - 10)^2 \text{ or } \psi = C\,10^{-0.025\,(H-10)^2}.$$

All researches into the structure of the universe must start from the knowledge of the function $A(m)$, the number of stars of a given magnitude. In practice we do not as a rule use A, but $N(m)$, the total number of stars down to a given limit of brightness, and so related to the former that $dN(m) = A(m)dm$.

Our want of knowledge of $N(m)$ was mainly due until a short time ago to the deficiency of good photometric measurings of the fainter stars on account of which the value of the limiting brightness m for a given numbering was unknown. Only these last few years this want has been to some extent removed. In N. 18 of the *Groningen Publications* the measurings extant have been collected and discussed; the results thus attained form at present the only reliable and firm basis for researches into the structure of the universe.

For the whole of the sky $N(m)$ is at present known to the 14th or 15th magnitude. From the brightest stars down to about the 11th magnitude the function is almost linear and we may put $N(m) = C + 0.50m$; after this the increase is less rapid. We know that a linear function $N(m) = c + \lambda m$, independently of the function $\psi\,(H)$ determines the distribution of densities; this distribution is then expressed by $\Delta\,(r) = r^{-5\,(0.60 - \lambda)}$. If the linear function mentioned with the coefficient 0.50 held good for the whole range of magnitudes, then we might conclude that $\Delta\,r = r^{-\frac{1}{2}}$, i.e. our star-system is densest in the middle and decreases in density towards the outsides, about inversely proportional with $\sqrt{r}$. That the fainter stars increase less in number than indicated by the formula, shows

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1) On the luminosity of the fixed stars. Public. Groningen N. 11. p. 16, 19.

that on the outside of our system the density decreases in a degree even greater than $1 : \sqrt{r}$.

But as it is the universe has not to be regarded as one whole nor the stellar system as a globular mass. The Milky Way forms a girdle across the sky where the star-density is greatest and from where it decreases to both sides. In second approximation N is no longer a function of m only but of b as well, the galactic latitude. KAPTEYN has given this function in N. 18 of the *Groningen Publ.* in a tabular form: $N(m, b)$, generally as well as $N(m)$ for three different parts of the sky, for the galactic zone, for the vicinity of the galactic poles and for an intermediate zone between 20° to 40° galactic latitude. The analytical functions deducted there do not, however, give an easy insight into their numerical values and these may be represented about as accurately by simpler and more easily manageable functions. For the galactic zone a linear function suffices:

$$log\, N_0^{20} = \bar{9}.70 + 0.49\,(m - 7)$$

For both the other zones the addition of a quadratic term is required and we may put:

$$log\, N_{20}^{40} = \bar{9}.48 + 0.49\,(m - 7) - 0.007\,(m - 7)^2$$
$$log\, N_{40}^{90} = \bar{9}.40 + 0.47\,(m - 7) - 0.009\,(m - 7)^2$$

By these formulae the structure of the universe is determined as a figure of revolution, a flat disk, its axis at right angles with the Milky Way. The star-density depends on two co-ordinates, the distance to the central plane z and the distance to the axis $\sqrt{x + y^2}$, or in polar co-ordinates: on the galactic latitude b and the distance to the sun r. These formulae show that the density decreases from the centre to all sides, fastest in the direction z at right angles with the plane of the galaxy, slower to all sides of the same. In the galaxy itself there clearly exists a rapid and regular decrease of density with the distance, according to the law $r^{-0.55}$.

This conclusion, however, is *in direct opposition to the appearance of the galaxy*. We see the galaxy as a belt of more or less circular masses, patches and drifts designating a totally different structure. Progressing in the direction of such a star-cloud we first expect an increase on this side of the cloud and then a decrease on the other side, which differs absolutely from what KAPTEYN's result for the galactic zone leads to. The appearance of the galaxy shows too that the zone between +20° and —20° galactic latitude should by no means be treated as one whole. In that way parts of the universe of really great diversity of structure would be mixed up; the galactic zone consists partly of star-clouds, forming the real phenomenon of the Milky Way, partly of intermediate and adjacent celestial regions possibly agreeing in formation with the galactic poles. It may be necessary to take all these different parts together for arriving at an average representation of the distribution of the stars in space, but this is *obscuring* the especially striking *character* of this distribution, which shows in the aggregation of stars into clouds and drifts; and it is giving a false impression of the real Milky Way if the star-density is represented as a simple function of r and b.

In order to obtain a true representation we must go on to a 3$^{\text{rd}}$ approximation; treat the special parts of the zone, the great patches and drifts in the Milky Way individually, determine for them separately N as a function of m and derive from this the value of Δ as a function of r.

The investigation communicated here is a first attempt to determine these functions for some parts of the galaxy, particularly to see what conclusions may be drawn to this purpose from the available material. Three regions were chosen to this end:

1. part of the large, bright Cygnus-patch, reaching from β to γ Cygni and forming the largest and brightest patch on the northern sky; this was chosen partly on account of its peculiar position[1]), partly because HERSCHEL's gauges are most numerous here. As these do not as a rule reach farther than 36° declination, only the region below this was taken. For limits were chosen parts of parallels at 1°, and parts of declination-circles at 4^{m} distance from each other, following as well as possible a boundary line on EASTON's map[2]). The sketch in fig. 2 designates these limits (region A).

2. part of the other branch of the galaxy in Aquila and Sagitta between 10° and 20° declination; the boundary line was taken from my own maps of the galaxy. This region was taken on account of its being a characteristic part of the main-branch in 18^{h}—19^{h} RA., as well as because HERSCHEL and EPSTEIN[3]) found here their richest fields. For this region too a broken line was assumed for boundary, as sketched on fig. 3 (region A).

3. For comparison a region on the other side of the sky at 6^{h} RA. was investigated, though less minutely. There is less contrast here between well defined smaller patches and a background of faint diffused light, and owing to this phenomenon larger regions

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[1]) See EASTON, La distribution de la lumière galactique, p. 45. (Verh. K. A. v. W. VIII, N. 3. 1903).

[2]) loc. cit. Carte isophotique.

[3]) Mitth. der V. A. P. Vol. III, p. 118.

were used corresponding to the trapezia counted by SEELIGER, viz., the areas 0°—5°, 6^{h}40^m— 7^{h}20^m; 5°—10°, 6^{h}0^m—6^{h}40^m; 10°—25°, 5^{h}20^m—6^{h}40^m.

§ 2. Let us first consider how a structure such as is to be expected from the aspect of the patches of the Milky Way, must present itself in the distribution of the stars $N(m)$. The luminosity-function is $log\, \psi = c - \alpha H^2$, assuming $H = 0$ for a star of magnitude 10.0 at a distance corresponding with $\pi = 0''.1$. For the distance r we introduce a new variable x, so that $x = 5\ log\, r$ and $x = 0$ for $\pi = 0''.1$; so the scale of x corresponds to the scale of magnitudes. We assume an agglomeration of stars at the distance r_0, the density of which decreases to both sides of r_0 according to the law

$$\Delta = 10^{-\mu(x-x_0)^2}.$$

Then the number of stars of brightness m is represented by

$$A_m = \int_{-\infty}^{+\infty} \Delta(x)\, 10^{0,6x - \alpha(m-x)^2}\, dr.$$

From this we conclude

$$log\, A_m = (0.6 + 2\mu\, x_0)\frac{\alpha}{\alpha+\mu}\, m - \frac{\mu\alpha}{\alpha+\mu}\, m^2$$

$$= C - \frac{\mu\alpha}{\alpha+\mu}\left[m - \left(x_0 + \frac{0,3}{\mu}\right)\right]^2.$$

We put $\frac{1}{\mu} = \sigma^2$, $\frac{1}{\alpha} = \tau^2$, then

$$log\, A_m = C - \frac{1}{\sigma^2 + \tau^2}\,[m - (x_0 + 0,3\sigma^2)]^2$$

So A is just like Δ and ψ an exponential function, having the form of the law of errors. If we express the function Δ by $log\, \Delta = -\left(\frac{x - x_0}{\sigma}\right)^2$, then we may call σ the dispersion of this function. Because, for $x - x_0 = \pm\, \sigma$ the value of the function becomes $^1/_{10}$; so practically that is the limit. For the agglomeration of stars σ, the dispersion toward both sides gives us an idea of its size. In the same way when in the luminosity-function we substitute $\frac{1}{\alpha} = \tau^2$, τ is the dispersion of that function; for $H = \pm\, \tau$, the number of stars becomes $^1/_{10}$ of the maximum. As $\alpha = 0.025$ we have $\tau^2 = 40$ and $\tau = \sqrt{40} = 6.3$ magnitudes. Now the formula shows that the maximum of the function A lies in the cluster of stars, i. e. in that magnitude which is most numerous for the distance of the cluster; not in the centre, however, but $0.3\,\sigma \times$ size of the cluster farther away; this is owing to the increase of the sector-volume with the distance. The dispersion of the A is the square sum total of the dispersions σ and τ of the curves of density and luminosity; the accumulation of Δ is considerably lessened in the A.

Now the dimensions of the galactic clouds in the radius vector cannot be very great, the largest patches, which seem to be round, stretch across the sky from 15°—20°, and if they have just as much depth in the radius vector as breadth in the perpendicular plane then their depth must be about $^1/_3$ of their distance; for $r = {}^2/_3$ and $^4/_3$ x becomes -0.9 and $+0.6$. So σ^2 is not much greater than 1, whereas $\tau^2 = 40$. The dispersion of the luminosity-function is therefore of considerably more influence in A, than that of the density. The great diversity in the luminosity of the stars causes each aggregation of them to be reflected only very faintly and diffusedly in the distribution of their numbers over different magnitudes.

The function N has then the form $\int\limits_{-\infty}^{m} 10^{-\frac{1}{2}\beta(m-m_0)^2}\,dm$ and can be calculated numerically. The function $log\,N$ first goes straight upwards and then approaches asymptotically the logarithm of the total number of stars of the cluster; the maximum m_0 lies where the curve is at a distance of 0.3 below this maximum (curve 1 in fig. 1). Now another

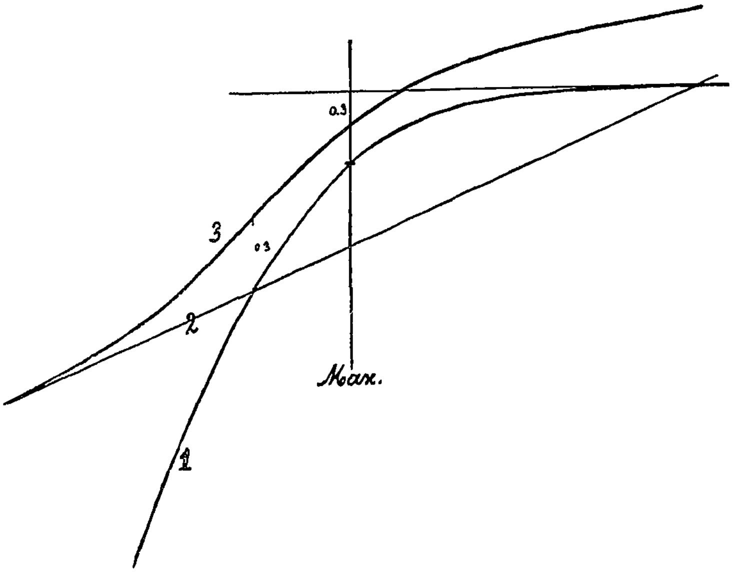

Fig. 1.

mass of stars, lying between us and the cluster, perhaps reaching even farther is added to the latter. The values of N for these stars, the same as for regions outside the Milky Way, may be represented by curve 2. So the total number of stars will progress like curve 3; first it will coincide about with curve 2, then it will continually rise above it, get a larger gradient and finally, past the maximum of the cluster, it will again go down to curve 2 and show a smaller gradient than this one.

If the density is the same along the whole of the radius vector the gradient of function N will be 0.60. If the density decreases regularly the gradient is < 0.60, if the density increases it is > 0.60. If this happens only in parts of the radius vector, the different gradients will be mixed up and strongly levelled, but still the general rule will be that a gradient below 0.60 indicates decreasing density and a gradient above 0.60 increasing density.

§ 3. The following material of star-countings could be used for our investigation:

1. The Bonner Durchmusterung. For magnitudes 6.55, 8.05 and 9.05, which we used as limiting ones, the photometric magnitude is known exactly. We could partly use the numbers of SEELIGER and STRATONOFF, partly they had to be counted anew. The total number 9.5 incl. was of no use, as its limiting brightness could not be sufficiently determined.

2. The gauges by W. HERSCHEL as published by HOLDEN in the 2nd vol. of the Washburn Observations. As nothing has been done during the whole of the 19th century to correct or complete HERSCHEL's gauges, they still form by the low limit to which they reach the most valuable, indeed an inestimable and indispensable material for researches into the structure of the universe. But owing to this, each want of homogeneity in these countings becomes an impediment to accomplishing such an investigation. Some parts of the Milky Way -- especially the Cygnus-patch at β Cygni — are very rich in counted fields, while not a single gauge occurs in the most northern parts of Cygnus towards Cassiopeia and Auriga. The gauges by J. HERSCHEL at the Cape are of no use for our purpose, as the results of the separate fields have not been published. The limiting magnitude has been computed by KAPTEYN on the photometric scale and found to be 13.9; through this determination only HERSCHEL's star-gauges have realized their full value.

3. The gap between the B.D. und HERSCHEL being considerable

it is of the utmost importance to have star-countings like **Herschel's** for intermediate limiting magnitudes as well. These have been executed by **Th. Epstein** at Frankfurt on the Main. From 1877 till 1888 he has gauged about 2700 fields distributed over the whole sky with a 6in. telescope. It is much to be regretted that the outcome of this interesting work has not been published and it is certain that the great value of such an investigation for the knowledge of the distribution of the fainter stars in the sky can only be realized by a detailed publication of its results. Mr. **Epstein** has kindly communicated to me the results needed for my researches and it will appear below how valuable they were.

4. The photographic *Carte du Ciel* as far as it has been published. This furnishes two sorts of data: for a brighter limit in the catalogue-plates, for a fainter one in the chart-plates. The limit not being the same for all parts of a plate owing to the curvature of the field — which has been found out to exist particularly in the plates of Oxford and Potsdam — these plates have always been taken *as one whole*; only the total numbers of stars on a plate or chart has been used as data.

In how far these data for the celestial regions examined are available, may be seen in the following sketches, fig. 2 and fig. 3, where the limits of the areas have been represented by broken lines. **Herschel's** gauges have been indicated by dots, those of **Epstein** by crosses.

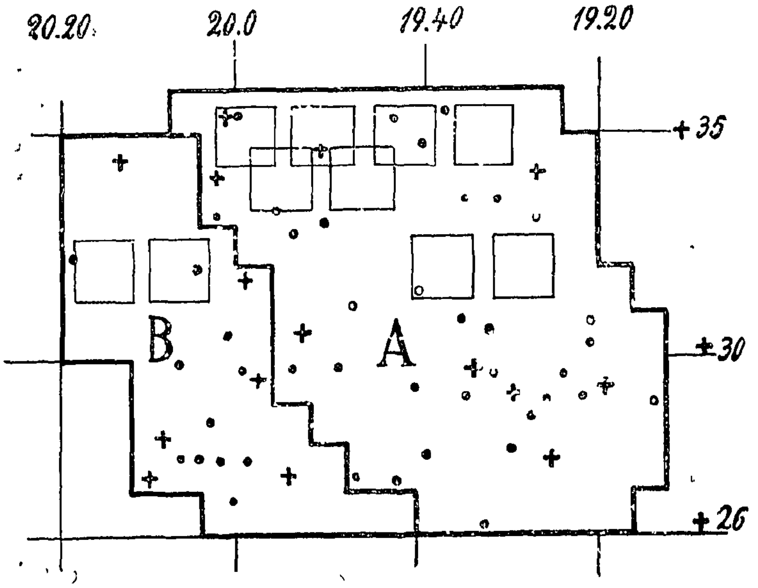

Fig. 2.

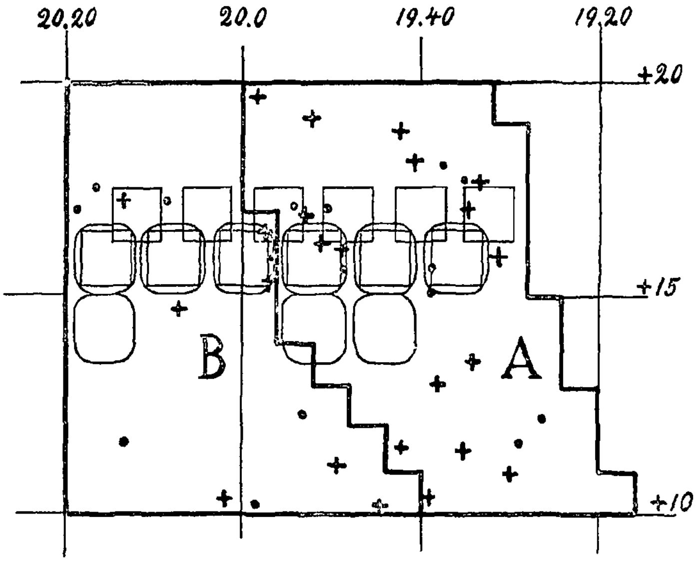

Fig. 3.

The fields of 2° length that are covered by catalogue-plates have been designated by small squares; those of which chart-plates are extant by bigger squares with rounded corners. For the Cygnus-region only catalogue-plates of Potsdam can be used, since by the strict equality of kind of plates and of time of exposure these form a homogeneous whole. The Oxford plates below 32° I dared not use; although the total number of stars is given for those plates that have not been completely measured, the diversity of kind of plates and of time of exposure made me fear a want of homogeneity that might be absolutely fatal here. For the Aquila-region I could only find catalogue-plates of the zones at 16° and 17° declination and chart-plates with centra at 14° and 16° declination, all of them from Bordeaux.

This is the weak point in our method of investigation: for with this irregular distribution of data the accidental irregularities in the distribution of the stars may not be sufficiently neutralized. The results for the number of stars N cannot be absolutely comparable unless they stand for the same region. If Herschel's or Epstein's gauges have been spread in a satisfactory number over the examined area then one may expect that from their average the irregularities have sufficiently disappeared. This is more doubtful for the plates of the *Carte du Ciel.* The outcome will have to show whether this want of identity of the regions examined has a more or less obnoxious influence.

§ 4. Now there remains to be determined the limiting magnitude for the numbers of stars per square degree found from each of these sources. For the *BD* it may be computed from SEELIGER's correction-formulae of the *BD*-magnitudes as a function of the star-density. For HERSCHEL it has already been determined by KAPTEYN to be 13.90. For the other sources it has been computed in the same way by using KAPTEYN's Table 1 in the *Gron. Publ.* N. 18 where *log* N is given as a function of b and m. For each of the sources used N was deduced for as many different parts of the sky as possible; for the numbers found and the b of each of these parts the table then furnished the corresponding $m =$ the limiting magnitude. So these magnitudes and with them the whole of our research is based on the photometric measures and on the countings on charts and star-plates that have been collected in Summary 11 of the work mentioned.

Before communicating the numbers thus found we must first face some difficulties. The irregularities of density are greater inside the Milky Way than outside; this is of the more importance as each division of the *Carte du Ciel* comprises only a narrow zone, and the average of both places where this zone crosses the Milky Way may differ considerably from the average of the whole of the galaxy. From this point of view it may seem desirable to use only the regions outside the Milky Way for the determination of the limiting magnitude.

On the other hand it is possible that there exists a systematic difference between the regions inside and those outside the Milky Way. The photometric scale on which KAPTEYN's tables are founded is a visual one, and his numbers though counted on photographs still indicate the numbers for visual magnitudes, supposing that the stars taken as standards have everywhere the same average colour as the great mass of the other stars. If the average colour of the fainter stars inside and outside the Milky Way is different — when the stars of small luminosity are on an average yellower than those of great luminosity, the average colour of the fainter stars in the Milky Way must be more blue than that of those outside — then N (m, b) must be different for visual and photographic m, and the limiting magnitude deduced from the photographic numbers with the help of KAPTEYN's tables will be found different for the inside of the galaxy and for the outside. In the hypothesis mentioned above m must be found greatest in the Milky Way. Such an error will be even more striking in the brighter patches than in the average galactic zone. By using all data, including those of the Milky Way itself, the error will be somewhat less than by using only the extra-galactic regions.

The visual countings by Herschel and Epstein are free from such an error, and this fact together with their homogeneity determines their great value even now that we have the photographic *Carte du Ciel.* But here another difficulty arises. Can it not be that the limiting brightness lies deeper in poor regions because in richer fields not all the fainter stars are counted? By the great influence of this error the numbers of the B.D. are, as we know, of no use for investigations such as ours. In so far as this error is occasioned by the fact that in rich regions many stars are neglected, it is a priori probable that it does not occur in simple *countings.* In the B.D. the stars were not counted but measured and that was the chief reason why very faint stars were taken in poor fields and omitted in richer ones. Kapteyn's research has already shown that in Herschel such a divergency from homogeneity cannot be indicated for certain.

Such a difference may also be due to the use of too low a power, while the emerging pencil of light is larger than the pupil; each increase of light in the field of view owing to a greater amount of stars then causes a contraction of the pupil and therefore a diminution of the actual aperture, hence a brighter limiting magnitude. In the introduction to vol. 8 of the Bonner Beobachtungen Schönfeld communicates the constants of the instrument used at Bonn for the northern Durchmusterung: Aperture 78 mm., magnifying power 9 times, diameter of the emerging beam of light $8^2/_3$ mm., diameter of the diaphragma often put before it 8 mm. "Dieser letztere ist immer noch grösser als die Pupillenöffnung der Beobachter unter mittleren, vielleicht selbst grösser als unter den günstigsten Umständen." So it is very likely that this circumstance has played some part in the B.D. The aperture of the instrument used by Epstein was according to the communications of the observer 16 cm., the magnifying power used was 80 times, so the diameter of the emerging beam of light was 2 mm., which certainly is always smaller than the pupil. Here of course there can be no question of such a systematic error.

The question may be put if an error like this can occur in the photographic *Carte du Ciel* owing to stars having been neglected in the richer fields. According to Prof. Scheiner's communications in the introduction to the 1st vol. of the Potsdamer *Catalog der photographischen Himmelskarte* p. xxx, this is quite impossible; the reverse should rather be feared, because in poor plates where there are many squares devoid of stars faint traces are more likely to escape observation than in rich plates where each square keeps the eye much longer. This cannot be, however, of great significance.

For the determination of the limiting brightness for Epstein 48 fields were used which were taken at random from the mass and are distributed over the whole sky. Each field gave a value for $N(m, b)$, from which the m was found for the given b. Divided according to the zones the averages are

b 40°—90°	12.56	(21 fields)
„ 20 —40	12.62	(10 „)
„ 0 —20	12.37	(17 „)

If from the latter one strongly diverging value is excluded (an excessively poor field between the branches of the Milky Way) this average rises to 12.49 (16 fields). So no systematic difference between the zones is indicated. As a general average we find the limiting brightness of all EPSTEIN's countings to be **12.51.**

The question may arise whether it would be better to leave that one diverging value out or perhaps to use only the regions outside the Milky Way, but that alters the value only less than 0.1 magnitude. From all EPSTEIN's material of course a much more accurate result might be found, but we may expect the number found here to be accurate to 0.1 magnitude.

The published plates of Potsdam are not distributed very regularly over the whole zone. After the b had been computed for the centre of each plate they were taken together in the order of RA. in greater or smaller groups covering about 10° galactic latitude, a little more in regions of which fewer plates exist. The plates being more numerous in 19^h RA. than elsewhere, many more plates were grouped together there not to give them too great a weight. The following table gives first the average values of b, then the average number of stars per plate, i. e. per 4 square degrees, next the number of plates of each group and finally the deduced limiting brightness m.

b	$4N$	n	m	b	$4N$	n	m
— 30°	194	9	11.79	+ 75	97	7	11.71
— 25	147	11	11.35	+ 83	86	10	11.59
— 14	209	8	11.27	+ 70	73	7	11.33
— 3	455	12	11.61	+ 55	100	5	11.53
+ 4	652	16	11.94	+ 41	137	7	11.69
+ 14	319	6	11.67	+ 24	316	5	12.07
+ 25	225	6	11.77	+ 14	423	5	11.94
+ 34	203	10	11.95	+ 4	723	29	12.04
+ 44	146	5	11.80	— 5	718	29	12.05
+ 54	132	7	11.84	— 12	442	10	11.88
+ 64	114	7	11.80	— 18	223	5	11.49

The 9 results belonging to the galactic zone give an average of 11.77, the 8 for galactic latitudes above 50° give 11.68; so there does not appear a systematic difference of any importance. The great abundance of stars in the Cygnus-regions happens to be so counterbalanced by the poverty of the Auriga-Perseus-region, that their average agrees with the general average. The average of all the plates together gives as

Limiting brightness of the Potsdam catalogue-plates **11.73.**

The catalogue-plates of Bordeaux have been measured belt-wise and in the first two volumes of the catalogue all plates with centres at 17° declination and all those with centres at 16° declination appear complete. In each belt we have grouped the plates together by fives in the order of RA. then computed for the average the b, the $4N$ and from this the m. Taking the results together again in 3 zones in the order of the b, we find:

b 40°—90°	Zone 17° $m = 12.13$ (14)	Zone 16° $m = 11.74$ (15)
20—40	11.92 (14)	11.71 (12)
0—20	11.76 (8)	11.72 (9)
	11.97 (36)	11.73 (36)

In the first belt there seems to be a considerable difference between the pole and the Milky Way, which does not occur at all in the second one. More striking still is the evident difference between the two adjacent belts (partly covering each other) although the instrument and the observers at the measuring-apparatus were the same. That a systematic difference does exist here, which does not find its origin in the sky, appears from the fact that the smaller fluctuations of richer and poorer fields in both belts run parallel with nearly always the same systematic difference. The explanation must be found in the observers at the measuring-apparatus having been unpractised in the beginning, so that they took the utmost care to discover each almost imperceptible star-spot and measure it; while after more practice they regularly left out the faintest traces as uncertain and took only those that were more definitely visible. The tendency to take into consideration even the faintest spots being likely to have been greater in poor regions than in rich ones, this explains at the same time the systematic difference between the pole and the Milky Way in the first belt. If this explanation is right then the later parts will show about the same results as zone 16° and we may assume as

Limiting brightness for the Bordeaux catalogue-plates **11.73.**

The plates of zone 17° might be reduced to the later method of measuring by subtracting an average of $^2/_{11}$ from the number of stars. But as we have only to do with plates of the Milky Way where the number counted is not too great it seems best to reduce also these plates of zone 17° with the same limiting brightness.

On the Bordeaux charts that have been published the total number of stars appearing on the cliché has been printed at the bottom; we assume these to stand for an area of 130′ length and breadth. These charts have not been distributed so regularly over the whole sky. We have arranged them, just as mentioned above for the Potsdam plates, in the order of the RA., taken them in groups of 2—7 plates, and computed the m for these groups. Thus we found for

9 groups	with b	40°—90°	$m = 13.03$
6 „	„	20 —40	13.30
6 „	„	0 —20	13.35

As a general average computed from 83 plates we find:

Limiting brightness of the Bordeaux chart-plates **13.20.**

An increase of the m towards the galaxy seems indicated.

§ 5. *In the following list the results of the countings in the* regions examined have been collected:

	Cygnus-region (area 85.3 □°)		Aquila-Sagitta-region (area 68.4 □°)		Monoc.-Taurus-region (area 385 □°)	
	Total	Per □°	Total	Per □°	Total	Per □°
BD 0—6.55	25	0.29	19	0.28	82	0.21
0—8.05	192	2.25	126	1.84	520	1.35
0—9.05	916	10.74	541	7.91	2800	7.27
All Stars	2958	34.68	1998	29.21	10833	28.14
	Per field	Per □°	Per field	Per □°	Per field	Per □°
HERSCHEL	269	5461 (32)	437	8871 (9)	97.6	1981 (37)
EPSTEIN		426 (9)		361 (16)		262 (18)
	Per plate	Per □°	Per plate	Per □°	Per plate	Per □°
Catal. pl.	729	182 (8)	710	177^5 (7)	549	137 (19)
Chart pl.	—	—	7528	1603 (5)	—	—

Now there remains the limiting brightness for the B.D. to be reduced to the photometric scale. From the total number per square degree we find:

	Cygnus	Aquila-Sagitta	Monoc-Taurus
D (Seeliger)	1.39	1.17	1.12
6.5 DM =	6.50	6.51	6.51
8.0 „ =	8.02	8.03	8.04
9.0 „ =	9.03	9.08	9.10

Putting together the numbers found as functions of the deduced limiting magnitudes and computing the *log N* and their gradients we find the following results to which Kapteyn's numbers for the average galactic zone have been added.

The values of the gradients $\frac{d}{dm}\, log\ N$ have each time been formed from the value of *log N* on the next higher and the next lower line.

Cygnus-region

m	*N*	*log N*	$\frac{d}{dm}\, log\ N$
6.55	0.29	9.46	
			0.59
8.07	2.25	0.352	
			0.67
9.08	10.74	1.031	
			0.46
11.73	182	2.260	0.46
12.51	426	2.629	0.68
			0.80
13.90	5461	3.737	

Aquila-Sagitta-region

m	*N*	*log N*	$\frac{d}{dm}\, log\ N$
6.56	0.28	9.45	
			0.54
8.08	1.84	0.265	
			0.60
9.13	7.91	0.898	
			0.52
11.73	177.5	2.249	0.49
12.51	361	2.557	0.65
13.20	1603	3.205	1.00
			1.06
13.90	8871	3.948	

Monoceros-Taurus-region

m	*N*	*log N*	$\frac{d}{dm}\, log\ N$
6.56	0.21	9.33	
			0.52
8.09	1.35	0.131	
			0.69
9.15	7.27	0.862	
			0.49
11.73	137	2.137	0.46
12.51	262	2.418	0.54
			0.63
13.90	1981	3.297	

Average galactic zone

m	*log N*	$\frac{d}{dm}\, log\ N$
6.55	9.467	
		0.51
8.08	0.239	
		0.50
9.10	0.745	
		0.48
11.73	2.013	
		0.46
12.51	2.375	
		0.45
13.90	3.003	

At first sight the gradients show an irregular up and down movement; past the 8th magnitude they rise, then they go down from the 9th to the 12th magnitude to a much lower value only to rise again rapidly after the 12th magnitude. This course appearing in all three regions the supposition seems obvious that it does not correspond to a real phenomenon in the sky, nor that it is the consequence of accidental error, but that it is caused by systematic errors in the *m*. It might be explained if the magnitudes in the vicinity of the 9th magnitude were all taken too low and in the vicinity of the 12th magnitude too high.

Now the limiting magnitudes for the B.D. have been found in a different way from those for the countings of the fainter stars; and KAPTEYN has already observed that the magnitudes accepted for the B.D. do not correspond with his tables; for these magnitudes he found the numbers diverge from the tables in the same sense as here, viz. they are 12°/₀ too great. With regard to this he says: "That the irregularity must be looked for not in the sky but either in the photometric determinations or in the countings, seems probable from the fact that for the most strongly diverging results the deviations for the zones 40—90, 20—40, 0—20 have the same sign and, speaking roughly, the same amount" [1]). If inversely one computes the limiting magnitude from the numbers reduced to 9.25 with the help of the tables, one will find not 9.25 but 9.36. Without looking into the reasons for this difference it is plain that for the sake of greater homogeneity it will be better to base the limiting brightness for the 9th magnitude also on KAPTEYN's tables. We shall therefore add 0.11 to all magnitudes of our table, standing for 9.0 B.D.

The question whether the magnitudes in the vicinity of the 12th are too high, is more difficult to answer. Taking into account that for the Potsdam catalogue-plates 11.0 on ARGELANDER's scale is meant for limit and that therefore the time of exposure, 5 min., was chosen in such a way as to first determine empirically the times of exposure for 7.0 and 9.0 B.D., after which the latter was once more enlarged in the same proportion, then one can expect at most 11.5 on the photometric scale. On the other hand KAPTEYN's tables correspond so well to the photometric measurings of the fainter stars that no error of great significance can be assumed here. Of course it would be of the utmost importance to control the accuracy of the deduced limiting brightness independently of KAPTEYN's countings. This might be done by finding among all the series of fainter stars that have been

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[1]) Groningen Publications N. 18 p. 39.

measured at HARVARD or by PARKHURST as comparison-stars for variables, those that occur in the Potsdam zone and by simply finding out which do and which do not occur in the Potsdam catalogue. This control I could not execute because it appeared that in the four volumes of Potsdam published till now there happens to be only one of the series of comparison-stars. As soon, however, as more volumes will be ready this course may be taken. For the present we have not a single definite indication that the magnitudes in the vicinity of the 12th are systematically too high. To remove the whole difference between the lower and the higher magnitudes an error of half a magnitude had to be assumed and this seems improbable. So for the 12th magnitude we keep to the magnitudes given above. After correction of the magnitudes in the vicinity of the 9th the gradients become

Cygnus		Aquila-Sagitta		Monoc-Taurus	
6.55		6.56		6.56	
	0.59		0.54		0.52
8.07		8.08		8.09	
	0.61		0.55		0.62
9.19		9.24		9.26	
	0.48		0.54		0.52
11.73	0.48	11.73	0.51	11.73	0.48
12.51	0.68	12.51	0.68	12.51	0.56
	0.80				0.63
13.90		13.20	1.00	13.90	
			1.06		
		13.90			

Here it must be observed that the course that is shown by these gradients and that completely depends on the values for the 12th magnitude, is found in the same way, with only a slight difference in the numerical value, from the photographic catalogue-plates and from EPSTEIN's countings. From this it appears that the accidental errors in our numbers, as consequences of the deficient identity of the fields for which they stand and of the irregularities in the distribution of the stars, are not so great as to obscure the result of our research. For what might seem doubtful before: whether the accidental irregularities would be sufficiently removed in our result, appears to be indeed the case as proved by two absolutely different sources corroborating each other.

The same holds good as well for the systematic differences between the photographic and the visual numbers of stars; these too cannot give rise to greater errors than the slight differences we find between the gradients computed from the magnitudes 11.73 and 12.51. Here it appears how valuable EPSTEIN's countings are; without this material it would have been impossible to state if the results from the photographic *Carte du Ciel* did not lead to an absolutely wrong outcome and false conclusions.

Now what are the conclusions that may be drawn from these numbers?

In the Cygnus-region as well as in that of Aquila-Sagitta the number of stars down to the 9th magnitude increases more rapidly than in the average galactic zone (gradient 0.50). Granting the accidental uncertainty in these numbers to be great on account of the smallness of the areas used still the difference will seem real; by further investigation it has to be decided whether the same holds good for all bright parts of the galaxy. Past the 9th magnitude the gradient for Cygnus goes very low down, just as low as for the average zone, while the decrease goes less far for Aquila-Sagitta. Past the 12th magnitude the gradient increases rapidly to far over 0.60. From this it appears *that in the direction of the bright galactic patches the star-density decreases at first and then increases again at a greater distance so that there occurs a real star-clustering, the influence of which is not felt before the numbers after the 12th magnitude.* This cluster is separated from the dense mass of stars surrounding us by an intermediate poor region that is especially perceptible in Cygnus. The increase is still greater in the Aquila-drift than in Cygnus; evidently the cluster is denser there. On the Monoceros-Taurus-side of the galaxy there occurs an increase in the gradient past the 12th magnitude but it does not or hardly reach over 0.60; so here there seems to be after a poor region a very slight, hardly perceptible aggregation.

In the same way we have also treated some regions of the sky on the eastern boundary of the regions in Cygnus and Aquila, so that the first is situated between the two branches, the other outside the Milky Way. Their limits have also been indicated in fig. 2 and fig. 3, where they form the regions *B*. The last named region has been examined because in HERSCHEL as well as in EPSTEIN and in the Bordeaux plates the richest fields are not found in the middle of the galaxy, but towards the eastside and because they even continue till outside the bright light of the Milky Way. The results of these countings and computations have been put down in the following tables:

Cygnus between the branches

m	N	$\log N$	$\frac{d}{dm}$
6.56	0.25	9.40	
			0.62
8.08	2.22	0.346	
			0.59
9.22 1)	10.48	1.020	
			0.46
11.73 4)	150.	2.176	0.41
12.51 2)	326.	2.513	0.46
			0.48
13.90 2)	1522.	3.182	

Aquila-Delphinus

m	N	$\log N$	$\frac{d}{dm}$
6.56	0.32	9.51	
			0.49
8.10	1.87	0.272	
			0.52
9.28 1)	7.59	0.880	
			0.50
11.73 5)	124.	2.093	0.51
12.51 3)	326.	2.513	0.61
13.20 6)	978.	2.990	0.90
			1.11
13.90 3)	5826.	3.765	

So the Aquila-Delphinus-region to the east of the galaxy has the same structure as this galaxy itself. The gradients are the same; after a continually decreasing density in the beginning they increase rapidly after the 12th magnitude. The density is everywhere somewhat less than in the central parts; so this region must be regarded as part of the body of the Milky Way, an outside part where the stars are less densely aggregated. A quite different picture gives the Cygnus-region between the branches of the Milky Way. It is hardly less rich in bright stars than the bright patch itself — compare with this **Argelanders**'s remark that in the B.D.-stars the bifurcation of the Milky Way is hardly perceptible —; on the other hand it lacks the increase of density past the 12th magnitude. This proves still more plainly that behind a region of stars, getting thinner with the distance and stretching over the whole breadth of the Milky Way, there occurs in one direction a dense star-cluster, which forms the bright Cygnus-patch, while we do not see the galactic light in the adjacent region where no such cluster occurs; this is the dark stroke between the two branches.

It does not seem advisable to draw still farther-reaching conclusions from this first material. It appears that with the stars as far as the 13.9 magnitude we only just reach into the greater star-clusters

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1) Already corrected with 0.11.

2) **Epstein** 6, **Herschel** 11 fields.

3) **Epstein** 7, **Herschel** 7 fields.

4) 2 plates.

5) 5 plates.

6) 4 plates.

forming the Milky Way, and in order to ascertain more about their structures and distances we have to go on to still lower magnitudes. That is why we do not venture here a comparison between our numbers and the light of the Milky Way. We only want to observe that the views based on former investigations have been rather contradicted than corroborated by this research. What has been found here indicates *that no organic relation exists between the great mass of stars of the* 9^{th} *magnitude and perhaps as far as the* 11^{th}, *and the star-clusters forming the Milky Way.* Before putting this down as a certainty, however, it is desirable that we should wait till we have more material available.

The completion and the publication of the photographic *Carte du Ciel* promises important results; it will be some time, however, before the charts fully cover the regions that are to be examined. But however much may be expected from a systematic treatment of the thus completed material, through combining the B.D., the catalogue-plates and the chart-plates of the *Carte du Ciel* and the star countings by HERSCHEL and EPSTEIN for different parts of the sky, still there remains the lack of homogeneity and of exact identity of the celestial regions for which these numbers stand. Another time I hope to describe a method free from these drawbacks.

Physiology. — "*The permeability of red blood-corpuscles in physiological conditions, especially to alkali- and earth alkali metals*[1]."

By Prof. H. J. HAMBURGER and Dr. F. BUBANOVIĆ (Croatia).

Introduction.

In a former communication one of us[2]) has demonstrated by means of quantitative chemical determinations that red blood corpuscles are in both directions permeable to Ca. At the same time the conditions were investigated under which this permeation took place. We have now extended our investigations to other Kations viz. magnesium, potassium, and sodium, and have finally connected with it the question whether, under the same physiological conditions under which the permeation of calcium, magnesium, sodium and potassium, was inves-

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[1]) More explicit communications on this subject will appear in the *Archives Internationales de Physiologie* publ. par LÉON FREDERICQ.

[2]) On the Permeability of blood cells to Calcium. These Proceedings of March 27 1909. See also a more detailed account in the Zeitschrift für Physikalische Chemie. Bd. 69, S. 663, 1909. (Festband f. Arrhenius).