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good deal more than those obtained formerly when the undivided figures were simply multiplied. But on the other hand there are other cases, particularly those in which a metasubstituted substance is nitrated, where this calculation does not agree with the experiment by a long way. If we take into account the figures of proportion for the single positions we obtain as a rule a much better approach to the figures observed by means of the products than with the sums, even in the case where the two substituents present are unequal, when HUISINGA's method of calculation cannot be applied. The proof thereof is laid down in the subjoined table which gives the figures of proportion in which the isomeric nitroderiva tives are formed from the substances at the top of the columns, with the figures obtained from both the sums and the products.

	Cl : Cl ortho	Cl : Cl meta	Br:Br ortho	Br:Br meta	$CO_2H:CO_2H$ ortho	$CO_2H:CO_2H$ meta
found	7 : 93	4 : 96	18.3 : 81.7	4.6 : 95.4	49.5 : 50.5	3.1 : 96.9
product	18 : 82	9 : 91	23 3 : 76.7	13 · 87	82 : 18	*10.6 : 89 4
sum.	18 : 82	15 : 85	23.3 : 76.7	19 : 81	55.6 : 44.4	*38 : 62

*totalquantity byproduct

	CO_2H : Cl ortho	CO_2H · Cl meta	CO_2H : Br ortho	CO_2H : Br meta
found	16 0:84.0	8.7 : 91.3	19.7:80.3	11.4:88.6
product	17.7:82 3	17.7:82.3	23.3.76.7	23.3:76.7

A fuller account of this investigation will appear in the Recueil.

Amsterdam, org. lab. Univ. 1906.

Astronomy. — *"The relation between the spectra and the colours of the stars."* By Dr. A. PANNEKOEK. (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN).

(Communicated in the meeting of September 29, 1906).

The close relation between a star's colour and its spectrum has long been known. The stars of the 1st, 2d and 3d types are usually called the white, the yellow and the red stars, although accurately spoken the colour of the so-called yellow stars is a very whitish unsaturated yellow colour and that of the so-called red stars is deep yellow mixed with very little red. In a paper read at Dusseldorf [1]) in

[1]) Die Farben der Gestirne. Mittheilungen der V. A. P. Jahrg. 10. S. 117.

1900 we showed that in the different glowing conditions the colours must succeed each other in this order. If for a given high temperature we accept the colour to be white, we find that with decrease of temperature the colours in the triangular diagram of colours make a curve which from white first goes directly to yellow of λ 587 but which, as the colour becomes deeper, bends towards the red and corresponds to light of greater wavelength. With increase of temperature, on the contrary, the line of colour runs from white to the opposite side, to the blue of λ 466.

Because the colours which are produced by white light after having been subjected to different degrees of atmospheric absorption, also follow about this same line, we may expect that the colours of the self-luminous celestial bodies will in general lie on this line or near it; they are determined on this line by one coordinate, one number. This renders it comprehensible why on the one hand the designation by means of letters and words, or the measurement with Zöllner's colorimeter, which produces quite different colours, has given so few satisfactory results, and on the other hand why the scale of Schmidt, who designates the colours by one series of figures, where 0 is white, 4 yellow, and 10 red has proved to be the best to work with. After this method has been drawn up the best and most complete list of stellar colours, published in 1900 by H. Osthoff at Cologne, in the A. N. Bd. 153 (Nr. 3657—58). This list in which the colours of all stars to the 5th magnitude are given, down to a tenth class of colour, and which was the fruit of systematic estimates during 14 years, enables us to accurately determine the relation between spectrum and colour.

In a former paper [1]) we remarked that we did not know where in the continuous series of spectra of the Oriontype and the first type we have to look for the highest temperature or at any rate the greatest luminosity. We may assume that it will be there where the colour is whitest; the spectral-photometric measurements, to which we have alluded in that paper, are still wanting, but for this purpose we can also avail ourselves advantageously of estimates of colour; this has been the reason for the investigation of which the results follow here.

In this case where we required a specification of the spectra, as detailed as possible, to serve as an argument for the colour, we have naturally used again Maury's classes. In order, however, to determine a mean colour for each class we must correct the colours

[1]) The luminosity of stars of different types of spectrum. Proceedings of June 30 1906 p. 134.

observed for two modifying influences, viz. the influence of the brightness and that of the altitude above the horizon. Quantitatively nothing is known about the values of these influences; experiments of OSTHOFF himself to determine the influence of the brightness have as yet yielded few results. Therefore we must derive them here from the material of stellar colours themselves, which serve for our investigation; this may be done in the very probable assumption that the real colour within each spectral class is an almost constant value and is independent of brightness.

§ 2. The stars of OSTHOFF's list which occur in the spectral catalogue of MAURY, were arranged according to their classes and then (excluding those which are marked *c*, *ac*, *C*, *P* or *L*, as was always done in this investigation) always taking together some classes, we classified them according to their brightness and combined their magnitudes and classes to mean values. These mean values must show the influence of the brightness on the colour; they are given in the following tables:

Classe III—VI

Mg.	Col.	
1.78	1.46	(5)
2.80	2.27	(6)
3.35	1.96	(5)
3.70	2.86	(7)
4.00	2.47	(8)
4.15	2.91	(7)
4.50	2.60	(9)
4.95	2.42	(11)

Classe VII—VIII

Mg.	Col.	
0.1	1.2	(3)
2.4	1.83	(6)
3.17	2.59	(7)
3.55	2.57	(6)
3.82	2.95	(6)
4.00	2.86	(5)
4.10	2.60	(7)
4.20	2.50	(5)
4.36	2.96	(5)
4.62	2.72	(4)
4.96	2.66	(5)

Classe IX—XII

Mg.	Col.	
1.0	2.7	(2)
2.69	2.97	(9)
3.18	3.06	(8)
3.65	3.73	(10)
3.85	3.40	(8)
4.10	3.69	(9)
4.29	4.17	(7)
4.65	3.79	(8)
5.10	3.34	(9)

Classe XIII—XIV

Mg.	Col.	
0.2	3.4	(1)
3.07	4.71	(7)
3.54	4.61	(7)
3.98	4.72	(9)
4 24	4.88	(8)
4.84	4.88	(8)

Classe XV

Mg.	Col.	
0.7	4.5	(2)
2.12	5.50	(6)
2.92	5.66	(9)
3.37	5.74	(9)
3.55	5.46	(9)
3.75	5.71	(8)
3.90	5.55	(10)
4.00	5 70	(7)
4.14	5.85	(11)
4.45	6.08	(6)
4.87	6.43	(7)

Classe XVI—XVIII

Mg.	Col.	
0.95	6.45	(2)
2 50	6.40	(6)
3.22	6.65	(6)
3.72	6.65	(4)
4.15	6.75	(6)
4.63	7.07	(7)
4.88	7.22	(9)
5.28	7.22	(8)

In all these series we clearly see an increasing deepening of colour with decreasing brightness. We have tried to represent the colour as a linear function of the magnitude; and by a graphical method we found:

Cl. III—VI	$c = 2.15 + 0.35$	$(m - 3)$
„ VII—VIII	$2.27 + 0.36$	„
„ IX—XII	$3.17 + 0.39$	„
„ XIII—XIV	$4.45 + 0.42$	„
„ XV	$5.47 + 0.39$	„
„ XVI—XVIII	$6.60 + 0.20$	„

Thus we find about the same coefficient in all groups except in the last. The value of the coefficients is chiefly determined by the difference between the observed colours of the very bright stars of the 1st magnitude and of the greater number of those of the 3d and 4th magnitudes. In order to make the coefficient of the last group agree with the others, it is necessary to assume for the apparent colour of α Tauri and α Orionis 5.6 instead of the real estimates 6,4 and 6,5. It does not do, however, to assume such a large error for these bright and often observed stars; therefore we must for the present accept the discordant coefficient of the red stars as real, although it is difficult at the present to account for it.

If now we combine the results of the five first groups by arranging the deviation of each observed value of c from the constant for the group (the value of c for $m = 3$), according to brightness and deriving thence mean values we find:

m	$c-c_3$	C_1	C_2	$O-C_1$	$O-C_2$
0.3	—1.03	—1.10	—0.91	+.07	—.12
1.6	—0.63	—0.54	—0.47	— 09	— 16
2.91	+0.02	+0.04	0.02	— 02	+ 04
3.73	+0.32	+0.31	+0.27	+ 01	+ 05
4.12	+0.48	+0.40	+0.39	+ 08	+ 09
4.73	+0.50	+0.52	+0.60	— 02	— 10

A linear relation $c = c_3 + 0{,}34\,(m - 3)$ yields the computed values given under C_2 and the differences obs.-comp. $O - C_2$. These are distributed systematically and show the existence of a non-linear relation. A curve, which represents as well as possible the mean values, gives the computed values C_1 and the differences, obs.-comp. $O - C_1$. For a greater brightness the curve gives a greater variation of the colour with the luminosity and for fainter stars a smaller one. In all the six groups, except the fifth and the sixth, we remark that the last values, which hold for the faintest magnitudes, show a decrease in the colour figures with regard to the preceding ones.

This phenomenon may be accounted for by the existence of the colourless perception of faint sources of light. In faint stars we do not see any colour at all; there the perception of colour disappears almost entirely and there remains only a colourless (i. e. whitish) impression of light. With stars which approach this limit, the impression of colour will be mixed up to a high degree with the colourless impression, and therefore they appear paler and will be indicated by a lower figure. As for the redder stars this colourless impression is relatively much weaker, the paleness of colour for these stars occurs only with a much less degree of brightness; in this manner we explain why the 5th and 6th groups do not show this decrease. Whether in these cases the phenomenon occurs with fainter stars cannot be decided because MAURY's spectral catalogue does not contain fainter stars.

For the practical purpose of reducing the observed colours to one brightness it is about the same which of the two relations is adopted, as long as we keep within certain limits of brightness, for instance between the magnitudes 1 and 5. To facilitate the reduction we have made use of the linear formula given above for the 5 first groups (down to class XV included) while for the redder classes 0,20 has been adopted as the coefficient of brightness.

To explain the long known phenomenon that the colour deepens with decreasing brightness as is shown in the tables on p. HELMHOLTZ in his *Physiologische Optik* has given a theory called "Theorie der kürzesten Linien im Farbensystem". In the diagram of colours in space, where each impression of light is represented by a point of which the 3 coordinates represent the quantities of the elemental colours, red, green, blue, the lines of equal colours are not straight radii through the origin, but curved lines which with increasing distance from the origin bend more and more towards the axes and so diverge more and more from one radius which is straight and represents the "Principalfarbe". Hence in the triangle of colours the points of equal colour diverge the more from the principal colour and run in curved lines towards the sides and the vertices as the triangle of colours is removed farther from the origin, and thus represents a greater brightness. HELMHOLTZ gives as principal colour a certain "yellow-white" to which with extremely great intensity all colours seem to approach. Therefore colours which lie on the blue side of this principal colour must become bluer by fading.

This does not agree with what we have found here, in the supposition that HELMHOLTZ's "yellow-white" is also yellow-white in our scale, i. e. is also represented by a positive number in SCHMIDT's scale. We also find here with the whitest stars that when they become fainter the colour becomes more yellow to just the same degree as with the yellower stars. Now the expression "yellow-white" is vague, but if we consider that what is called white in the scale of SCHMIDT is whiter, that is to say bluer than the light of Sirius, and that the solar light, the standard for white for ordinary optical considerations, if weakened to the brightness of a star, in the scale of SCHMIDT would be called 3 à 4 (Capella 3, 4), then the principal colour, if HELMHOLTZ's theory is true, instead of being yellow-white would still lie on the blue side of the Sirius light.

§ 3. After the colours had thus been reduced to the brightness 3,0, they had still to be freed from the influence of the atmosphere, which makes them redder. This cannot be done with the desired accuracy, because neither time nor altitude are given along with the observations. The influence at high and mean altitudes is probably very small, and the observer is sure to have taken care that most of the stars were observed at a proper altitude (for instance between 30° and 60°). Therefore this correction is only practically important for the few southern stars which always remain near the horizon; in these cases it will be possible to represent the variation of colour by a correction depending on the declination. Instead of the declination of the star we have taken the declination of the B. D.-zone which OSTHOFF has added to his catalogue.

For each spectral class we have determined mean colour-values for all stars north of the equator, and for the stars south of the equator we have formed the deviations from these class-means which then were arranged according to their declination and combined to mean values for groups of stars. We have excluded, however, those classes in which too few northern stars occurred, namely I, II and III.

The means found are:

Zone	Deviation	n.	Curve	Zone	Deviation	n.	Curve
0°0	+0.56	5	+0.05	—9°0	+0.14	5	+0.26
—1 7	+0.35	4	+ 06	—10.2	+0.35	4	+ 32
—3.3	—0.17	6	+ 09	—13.2	+0.33	6	+ 57
—5.0	+0.50	5	+ 12	—15.0	+1.17	6	+ 79
— 6.6	+0.22	4	+ 17	—18.2	+0.93	6	+1.32
—8.0	—0.05	5	+ 22				

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Through these values we have drawn a curve which from the equator towards the southern declinations ascends steeper and steeper and which gives the values of the last column. According to this curve we have applied the following corrections, for

zone	1°	2°–5°	6°–8°	9°–10°	11°	12°	13°	14°	15°	16°	17°	18° South
neg. corr.	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,1	1,3

We may assume that by these corrections the variation of colour due to atmospheric absorption has at least for the greater part been eliminated.

§ 4. After the two corrections (§ 2 and § 3) had been applied we could determine for all spectral classes the mean values of the colour; they are given in the following table. Class XV was again subdivided into 3 classes according as the spectrum agreed with α Bootis (*A*) or with α Cassiopeiae (*C*) or was not accompanied by any such remark; the result shows indeed that here class *C* is considerably redder than class *A* while the *B*'s lie between the two.

Class	Colour	Number	Class	Colour	Number
I	2.47	6	XII	3.68	17
II	2.36	10	XIII	4.12	13
III	2.30	9	XIV	4.45	12
IV	1.94	14	*XIV*	5.09	9
IV	1.62	10	XVA	5.18	18
V	2.11	9	XVB	5.35	26
VI	2.16	10	XVC	5 55	31
VII	2.27	23	*XV*	6.34	5
VIII	2.37	34	XVI	6.47	17
IX	2.64	20	XVII	6.80	15
X	3.11	14	XVIII	6.74	15
XI	3.40	9	XIX	6.67	6
XI	3.41	4			

The deviations of the separate values from these mean values give, as a measure for the accuracy of the results, for the mean error of a colour-number, $\sqrt{0,20} = 0,45$; the real accuracy will be greater, however, and the mean error smaller because in these values are also included the errors of the adopted corrections for brightness and declination, the errors which may have been made by MAURY while classifying each star in a definite class, and also the real deviations of the single-stars from their class-means.

With 9 stars (out of 355) the deviation exceeds a unit of colour; the reduced colours are here:

β Can. maj.	III	1,2	ε Hydrae	XIII	5,2	η Persei	XV*B*	6,8
o_1 Cygni	IX	1,4	μ Persei	XIV	5,5	11 Urs. min.	XV*B*	6,6
δ Delphini	IX	3,8	o_2 Cygni	*XIV*	6,5	5 Orionis	XVII	7,9

In this investigation we have, as it was said before, excluded the *c*- and *ac*-stars, the *L* (bright lines), the *P* (peculiar spectra) and the *C* (composed spectra). It is important to examine the *c* and the *ac*-stars among them more closely in order to see whether they show a distinct difference in colour from the *a*-stars of the same class-number. In the mean 11 *ac*-stars give a deviation of + 0,1 (from + 0,5 to — 0,3), and 12 *c*-stars + 0,7; so these last ones are a little redder than the *a*-stars. Here, however, the great individual deviations are very striking; the extreme values are:

ϱ Cassiop XIII + 2,5; χ_2 Orionis III + 1,8; 4*H* Camelop *VI* + 2,0; 3*H* Camelop *VI* + 1,5; η Leonis VII — 0,3; β Orionis VI — 1,2,

The differences are very great, but no regularity can be detected.

§ 5. The results found solve a problem which in my former paper remained unsolved, namely where in the continuous series of spectral classes shall we have to look for the maximum of radiating power. The colour-numbers show very distinctly a fall in the first classes, a *minimum* between the 4[th] and the 5[th] class and then a continual rise. The stars which in order of evolution directly follow on γ Orionis (μ Aurigae, μ Hydrae, *u* Herculis) have the whitest colours; both the earlier and the later stages of evolution are yellower; classes I and II agree in colour best with class VIII. Therefore, in so far as we are entitled to derive the entire radiation from the colour, the maximum of radiating power lies between the 4[th] and the 5[th] class.

The mean colour-numbers for each of the groups formed before are:

Cl. I—III	2.35
IV—V	1.87
VI—VIII	2.30
IX—XII	3.20
XIII—XIV	4.58
XV	5.43
XVI—XIX	6.66

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Is it possible to derive from these numbers, even though only approximately, values for the radiating power per unit of surface? The two influences mentioned above which determine the colours of the self-luminous celestial bodies may be subjected to a calculation, if we disregard the specific properties of the composing substances and treat them as abstract theoretical cases. In other words we can investigate the radiation of a perfectly black body and in the absorption neglect the selective absorption in lines and bands in order only to examine the general absorption. As a first approximation this may be deemed sufficient.

In this calculation we have made use of the measurements of A. KONIG on the relative quantities of the elemental colours red, green and blue as functions of the wavelength in white sunlight. If for an other source of light we know the relation of the brightness with regard to the former source as a function of the wavelength, we can calculate the quantities of the red, green and blue in this second source of light. If we call the numbers of KONIG $R(\lambda)$, $G(\lambda)$, $B(\lambda)$, which are chosen so that

$$\int R(\lambda)\, d\lambda = 1000 \qquad \int G(\lambda)\, d\lambda = 1000 \qquad \int B(\lambda)\, d\lambda = 1000$$

and if $f(\lambda)$ represents the brightness of another source of light, then

$$\int f(\lambda)\, R(\lambda)\, d\lambda \qquad \int f(\lambda)\, G(\lambda)\, d\lambda \qquad \text{and} \qquad \int f(\lambda)\, B(\lambda)\, d\lambda$$

represent the quantities of R, G, and B occurring in this light. As the impression of brightness of a source of light is almost proportional to the quantity of red, this calculation gives at the same time a measure for the optical brightness.

The radiation of a black body may be represented by:

$$\lambda^{-\alpha}\, e^{-\frac{c}{T\lambda}}\, d\lambda$$

where T is the absolute temperature and α and c constants. For two sources of light of different temperatures the relation of the intensities is:

$$f(\lambda) = e^{-\frac{c}{\lambda}\left(\frac{1}{T} - \frac{1}{T_0}\right)} = e^{\frac{b}{\lambda}} = 10^{\frac{b'}{\lambda}},$$

if $b = c\left(\frac{1}{T_0} - \frac{1}{T}\right)$ and $b' = 0.43\, b$. As unit for λ we adopt 0,001 mm; T_0 is supposed to be given, then b' is a function of the variable temperature T only and may be called the degree of glowing with regard to the glowing of a body at a temperature T_0. If we adopt for b' different values ($c = 15000$ about) [1]), we can calculate for each of them the brightness and colour of the light, as well as the temperature T. We then find for the degrees of glowing $+1$, 0 and -1

$$\begin{array}{rrrr} b' = +1 & 69200\,R + & 68100\,G + & 175800\,B \\ 0 & 1000\,R + & 1000\,G + & 1000\,B \\ -1 & 17{,}7\,R + & 15{,}7\,G + & 6{,}3\,B \end{array}$$

If we represent the colour contained in a total quantity of light of 1000 by the quantities R, G, B and the brightness by magnitudes, we shall find for

$$\begin{array}{lll} b' = +1 & Col. = 221\,R + 218\,G + 562\,B & Br. = +\,4{,}6\ Mg. \\ b' = -1 & Col. = 445\,R + 396\,G + 160\,B & Br. = -\,4{,}4\ Mg. \end{array}$$

Thus the first colour may be described as a mixture of 654 white and 347 of a blue consisting of 3 R and 344 B, hence corresponding in tint to λ 466; the second colour is a mixture of 480 white and 521 of a yellow consisting of 285 R and 236 G, hence corresponding to the wavelength λ 587. A degree of glowing $b' = -2$, corresponding in colour almost with the light of petroleum, involves a decrease in brightness of 8,6 magnitudes.

For the calculation of the atmospheric absorption we have assumed that the general absorption in a gas is inversely proportional to the fourth power of the wavelength. For a layer of gas adopted arbitrarily, which after a comparison with MULLER's spectral-photometric measurements appeared to correspond to 1,05 atmosphere, we have calculated $f(\lambda)$ and thence found for the remaining quantity of light, the initial quantity being $1000\,R + 1000\,G + 1000\,B$:

$$783\,R + 771\,G + 571\,B,$$

or reduced to 1000 as the sum,

$$368\,R + 363\,G + 269\,B;$$

the brightness is then 0,783 of the original brightness or is diminished by 0,27 magn.

The colouring due to the absorption by 1.05 atmospheres is almost equal to that brought about by a diminishing of the degree of glowing of $^1/_3$. For the latter yields

$$257\,R + 248\,G + 184\,B$$

hence when reduced to a sum of 1000

$$372\,R + 361\,G + 267\,B$$

1) In the paper read at Dusseldorf (see note p. 292) wrong temperatures are given because the difference between b and b' was overlooked. The temperatures 16000°, 7500°, 5000°, 3750°, 3000° C do not differ inter se 1, but only 0.43 in degree of glowing.

which is nearly identical with the value above. Here, however, the brightness is diminished to 0.257 of the original, hence by 1.48 magnitude.

Therefore it appears here that these two different causes produce similar colours, but that they correspond to an entirely different decrease of brightness. When comparing the two we may say that atmospheric absorption is more apt to redden, a decrease of temperature more apt to fade the light. Therefore it is impossible to derive the radiating power from the colour only, as we do not know to what degree each of the two influences, temperature and absorption, is at work in the different spectral classes. Perhaps that one day accurate spectral-photometric measurements will enable us to separate the two influences, for they give a different distribution of intensity over the spectrum. For the log. of the brightness of different λ with regard to λ 500 we find

	$\lambda =$ 650	600	550	500	450	400
with abs. 1.05 atm.	+ 0.114	+ 0.083	+ 0.051	0.000	−0 084	−0.231
with glowing — $^1/_8$	+ 0.154	+ 0.111	+ 0.061	0.000	−0.074	−.0166

For the latter the decrease in intensity from the red to the violet is more regular, for the former the decrease is slower for the greater and more rapid for the smaller wave-lengths.

These calculations show that it is not strictly true that, as has been said in the preceding paper, a redder colour must necessarily involve a smaller radiating power. Where we have two influences which in different ways bear on the colour and the brightness, the possibility exists that a redder colour may be accompanied by a greater radiating power, namely when one source of light has a much higher temperature and at the same time a greater atmospheric absorption than the other. An increase of the degree of glowing of + $^1/_8$ combined with an absorption of 2 atmospheres gives such a case according to the figures given above.

Herein we have therefore a new possibility to account for the peculiarities found in the *K* stars, namely by assuming that, as compared with the *G* stars, they have a much higher temperature, which causes a stronger radiation, and which by very strong atmospheric absorption, is only little faded but greatly reddened. We must add, however, that this explanation seems little probable to us as the band-absorption, which begins at the *K*-stars and which is characteristic for the *M* stars (the 3rd type) indicates a lower temperature.